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All communications should be addressed to the Editor.

No. 144. DECEMBER, 1922. VOL. XXVI.

Notices of the Royal Aëronautical Society.

Election of Members.

The following Members were elected at a Council Meeting held on November 21st:—

Fellow.—C. R. Fairey.
Associate Fellows.—H. J. Mackintosh and S. E. Taylor.
Member.—Major H. G. Brackley, D.S.O., D.F.C.
Foreign Member.—J. Parker van Zandt.
SCOTTISH BRANCH.—*Associate Member.*—F. O. W. J. Buchanan.

R.38 Memorial Prize.

On the next page will be found the full regulations for the annual award of the R.38 Memorial Prize for papers on airships. It will be noticed that for the coming year's award synopses of papers should reach the Secretary on or before the 31st of the present month and the full paper not later than March 31st, 1923.

The Council have given the commission for the design of the memorial tablet to those who were lost in the airship, to be placed in the Library, to Mr. Paul Cooper, who has already presented preliminary sketches and is now engaged upon the design.

Library.

The Council are glad to announce that through the kind assistance of Lord Weir a successful application has been made to the authorities of the Carnegie United Kingdom Trust for a grant towards the purchase of volumes from a valuable collection of early historical books on aeronautics which would otherwise have been sold to an American collector. Owing to the generosity of the Carnegie Trustees it is possible to fill a large number of the gaps in the Society's collection, as well as to obtain a number of modern textbooks. When the purchase is concluded the Society's Library will be indubitably the most complete and representative collection of books on all aspects of aeronautics in this country.

At the request of the Carnegie Trustees the books in the Library will in future be available to students applying through the Central Library for Students.

Students' Visit.

A visit to the collection of aeronautical exhibits at the Science Museum, Exhibition Road, South Kensington, has been arranged for Saturday afternoon, December 16th, when Mr. Davy, in charge of the aeronautical section of the

Museum, will conduct the party round. Students who wish to take advantage of this opportunity will meet at the entrance to the Southern Galleries of the Science Museum at 3.0 p.m.

Technical Discussion.

The first "Technical Discussion," open to all technical members of the Society, was held in the Library at 3.0 p.m. on November 30th, when Mr. H. Glauert's paper, "Theoretical Relationships for the Lift and Drag of an Aerofoil Structure," was discussed. The discussion was adjourned to 3.0 p.m. on Wednesday, January 10th, 1923.

Additions to the Library.

The following books have been received and placed in the Library:—"Direction and Position Finding by Wireless," by R. Keen; "Index to Periodicals K. Science and Technology," by the Library Association; "Our Future in the Air," by General P. R. C. Groves; "The Aerofoil and the Screw Propeller," by F. W. Lanchester; "L'Aeronautique," by Comte de la Vaulx; "The Internal Combustion Engine" (Vol. I.), by H. R. Ricardo; "Aircraft Steels and Materials," by various authors; "Sir Walter Raleigh and the Air History," by H. A. Jones; "The Internal Combustion Engine," by Major Wimperis; Transactions of the Newcomen Society (Vol. I.).

Forthcoming Arrangements.

Dec. 14, 7.0 p.m.—Society's Library, Students' Section. "Navigation of Aircraft," by Mr. A. P. Rowe. Chairman, Sir A. Whitten Brown.

,, 16, 3.0 p.m.—Visit to Science Museum. Students' Section. Meet at Museum entrance, Exhibition Road, S.W.7.

1923.

Jan. 4, 5.30 p.m.—Royal Society of Arts. Herr Junkers, "Metal Aeroplanes."

,, 10, 3.0 p.m.—Society's Library. Adjourned Technical Discussion.

,, 11, 3.0 p.m.—Royal Society of Arts. *Juvenile Lecture.* Mr. R. A. Frazer, "Model Aircraft."

Students' Smoking Concert.

It has been suggested that a smoking concert should be held by the Students in London early in the new year, and it is proposed to discuss the matter with a view to coming to some decision at the Students' Meeting on the 14th of this month. It is hoped that Students will make a special point of attending this meeting, or if unable to do so will write to the Honorary Secretary saying whether or not they are in favour of the proposal.

W. Lockwood Marsh, *Secretary.*

R38 MEMORIAL PRIZE.

The Council of the Royal Aeronautical Society have decided to institute forthwith from the funds of the R.38 Memorial Research Fund an annual prize for a technical paper on Airships. The regulations covering the award of this prize are given below, from which it will be seen that the date for the receipt of the names of intending competitors for the first award is December 31st, 1922, while the papers themselves must reach the Secretary on or before March 31st, 1923.

REGULATIONS.

From the income of the above fund a sum of twenty-five guineas will be offered annually as a prize for the best paper received by the Royal Aeronautical Society, on some subject of a technical nature in the science of aeronautics. Other things being equal, preference will be given to papers which relate to airships.

The prize is open to international competition. The Royal Aeronautical Society retain the right to withhold the prize in any year, if it is considered that no paper is of sufficient merit to justify an award.

Intending competitors should send their names to the Secretary of the Royal Aeronautical Society, 7, Albemarle Street, London, W.1, on or before December 31st of each year, with such information in regard to the projected scope of their papers as will enable arrangements to be made for their examination. The closing date for the receipt of papers will be March 31st in each year.

Papers should in all cases be typed, and a copy should be retained by the author as the Society can take no responsibility for the loss of copies submitted to it.

Successful papers will become the absolute property of the Society and will in most instances be published in the Society's Journal. In regard to unsuccessful papers, the Society retains the right of publication in its Journal, but in each case will notify the author, shortly after the award, whether it intends to exercise this right; if not, the author will be free to publish elsewhere. A signed undertaking must accompany each paper to the effect that publication has not already taken place and that the author will not communicate it elsewhere until the Society's award is published. Due acknowledgment must be made by the author of the source of any special information.

THE ELASTIC CONSTANTS OF SPRUCE AS INFLUENCED BY MOISTURE.

BY H. CARRINGTON, B.SC. (VICT.), M.SC. (TECH.), A.F.R.AE.S.

Introduction.

Previous investigations on the effect of moisture on the elastic constants of spruce have been confined to its effect on Young's modulus in the direction of the grain. The most important work in this connection is probably that of H. D. Tiemann,* who deduced curves for pine spruce and chestnut showing that Young's modulus along the grain decreased as the moisture increased up to the fibre saturation point, and that further addition of moisture had little or no effect on the value of the modulus. Tiemann thus found that the modulus decreased to a minimum at the fibre saturation point and then remained nearly constant and independent of any further addition of moisture. The reason for the independence of the modulus on the moisture beyond the fibre saturation point becomes evident if it is considered that when the fibres are saturated, any further moisture added is retained in the cells and in all probability cannot further affect the stiffness and strength of the wood.

It is proposed in this paper to give the results of a series of experiments made to determine the effect of moisture on the elastic constants of spruce, the constants being determined on the assumption that the spruce has three planes of elastic symmetry. Some consideration will also be given to the relation between shrinkage and moisture and to the effect of continued drying on the shrinkage and moisture and consequent possible damage to the cell walls.

Notation.

Let the direction of the grain of the tree be denoted by ZOZ, the direction perpendicular to this and intersecting the axis of the tree by XOX and the direction normal to ZOZ and XOX by YOY. Thus, the direction XOX is normal to the annual layers, and the direction YOY tangential to them. If, therefore, three mutually perpendicularly lines be considered lying in the above three directions and intersecting at a point O situated some distance from the pith of the tree, the lines are the intersections of the three mutually perpendicular planes of symmetry OYZ, OZX and OXY at the point.

Corresponding with direct strain, in any one of the three principal directions, three elastic constants will be involved. For direct strain in the direction XOX will cause lateral strain in the directions YOY and ZOZ, and thus involve Young's modulus in the direction XOX and two values of Poisson's ratio corresponding respectively with lateral strain in the directions YOY and ZOZ. Thus, corresponding with the three principal directions there will be three principal values of Young's modulus and six principal values of Poisson's ratio.

Let E_x, E_y and E_z denote Young's modulus in the directions XOX, YOY and ZOZ respectively. Also let σ_{yz}, σ_{zy}, σ_{zx}, σ_{xz}, σ_{xy} and σ_{yx} denote the six values of Poisson's ratio where

σ_{yz}=(lateral strain in the direction ZOZ)/(longitudinal strain in the direction YOY).

Further consideration will show that there are also three principal values of the modulus of rigidity corresponding with shear strain along any of the three pairs of principal planes. Let M_{yz}, M_{zx} and M_{xy} denote these values of the

* U.S. Department of Agriculture, Forest Service Circ. 108, dated August, 1907.

modulus where M_{yz} . . . denotes the modulus of rigidity corresponding with shear strain along the two principal planes OZX and OXY, *i.e.*, along the directions YOY and ZOZ.

The Test Pieces and Experiments.

The specimens were prepared from one of four baulks of spruce obtained from the R.A.E. and representative of the good average spruce which was available for aeroplane manufacture early in the war. The baulks were well seasoned and free from knots and shakes. Typical microphotographs and an end view of one of the baulks have previously been published.* In these publications the baulks were lettered A, B, C and D and the experiments described in this paper were performed on baulk D.

The values of Young's modulus and Poisson's ratio were deduced from flexure experiments. In the case of the determination of E_z and either σ_{zx} or σ_{zy} the test pieces were about 13in. long, 1in. wide and $\frac{1}{4}$in. thick, and were supported on knife edges 12in. apart, the load being applied to two other knife edges 8in. apart and symmetrical with respect to the outer knife edges. When the load was applied the portion of the beam between the inner knife edges was evidently under a constant bending moment and the constants were determined from measurements of the longitudinal and lateral curvatures over about 1in. lengths situated at the mid-lengths of the beams. When performing an experiment the bending moment was increased by equal amounts at a uniform rate, care being taken not to overstrain the pieces, and readings proportional to the curvature taken after every increase by a reflection method.† In this way two straight lines were constructed, the slopes of which represented the longitudinal and lateral curvatures respectively per unit couple. If the length of the beam was in the direction ZOZ and the width in the direction XOX the experiment yielded values of E_z and σ_{zx}, the latter being given by the ratio of the curvatures. The procedure in the determination of E_x and E_y, and the corresponding values of Poisson's ratio was exactly similar, but the specimens were shorter and their breadths and thicknesses proportionately smaller.

The moduli of rigidity were obtained from torsion experiments on prisms of rectangular cross-section. When a prism with its length in the direction ZOZ and its sides parallel to the two planes of symmetry OYZ and OZX is twisted about its longitudinal axis two values of the modulus are involved, *i.e.*, M_{yz} and M_{zx} and by making the ratio of the sides of the cross-section suitable it is possible to calculate one or other of the moduli with considerable accuracy. If the thickness of the piece is in the direction YOY then M_{zx} can be obtained with an error of less than 0.1 per cent. provided (breadth/thickness)$\sqrt{(M_{yz}/M_{zx})} > 3$.‡ Since the ratio $\sqrt{(M_{yz}/M_{zx})}$ is approximately unity the breadths and thicknesses of the pieces with their lengths in the direction ZOZ were made about $1\frac{1}{4}$in. and $\frac{1}{4}$in. respectively.

The values of M_{xy} were determined in a similar manner. The length of the prism was in the direction XOX and the breadth and thickness in the directions YOY and ZOZ respectively. In order that the error in the value of M_{xy} should be less than 0.1 per cent. then (breadth/thickness)$\sqrt{(M_{zx}/M_{xy})}$ should be > 3 and since $\sqrt{(M_{zx}/M_{xy})}$ was equal to about 5 then M_{xy} could have been calculated if the cross-sections had been square. The actual dimensions were about 1in. by $\frac{1}{4}$in. in the directions YOY and ZOZ respectively.

The moduli were deduced from torque twist curves, the twist being measured by the usual reflection method of affixing mirrors to the specimens and using telescopes and scales. More detailed accounts of the methods of determining the

* Phil. Mag., June, 1921, and May, 1922.
† Phil. Mag., Feb., 1921.
‡ Phil. Mag., June, 1921.

constants and the rate of loading the specimens will be found in the Phil. Mag. papers quoted.

Method of Varying the Moisture.

A curve connecting any particular constant with the moisture could have been obtained either by performing experiments on a number of pieces each with a different humidity or by using one piece and varying the humidity between each experiment. The former method was used by Tiemann, but it was decided to adopt the latter in this research in order to avoid the effect of variability in the eolotropy of different pieces. Seven pieces were accordingly prepared and seven series of experiments performed, one corresponding with each piece. The pieces were allowed to remain in the laboratory for about one month after preparation before the first experiments were performed. Their humidity was then about 12 per cent. After the first experiments had been carried out the pieces were dried in an electrically heated stove at a temperature of from 100 to 104 degs. C. for from 24 to 48 hours and tested immediately on leaving the stove, and at intervals until they had again absorbed about 12 per cent. of moisture from the atmosphere of the laboratory. They were then placed over water in a sealed vessel at normal temperature and removed for test at intervals. Immersion in water vapour raised the humidity to between 20 and 25 per cent., and in order to increase it well beyond the fibre saturation point the pieces were placed in water. Previous to testing at humidities beyond the fibre saturation point the pieces were suspended in the sealed vessel over water vapour for some time to retard evaporation and thus avoid internal strains and non-uniformity in the distribution of the moisture.

By the above procedure the moisture was varied from nearly zero to well over 100 per cent. and experiments were performed at humidities below the fibre saturation both before and after the pieces had been in water. The percentage moisture was defined in every case as

(weight when tested—weight when dry) (weight when dry) × 100.

Results of Experiments.

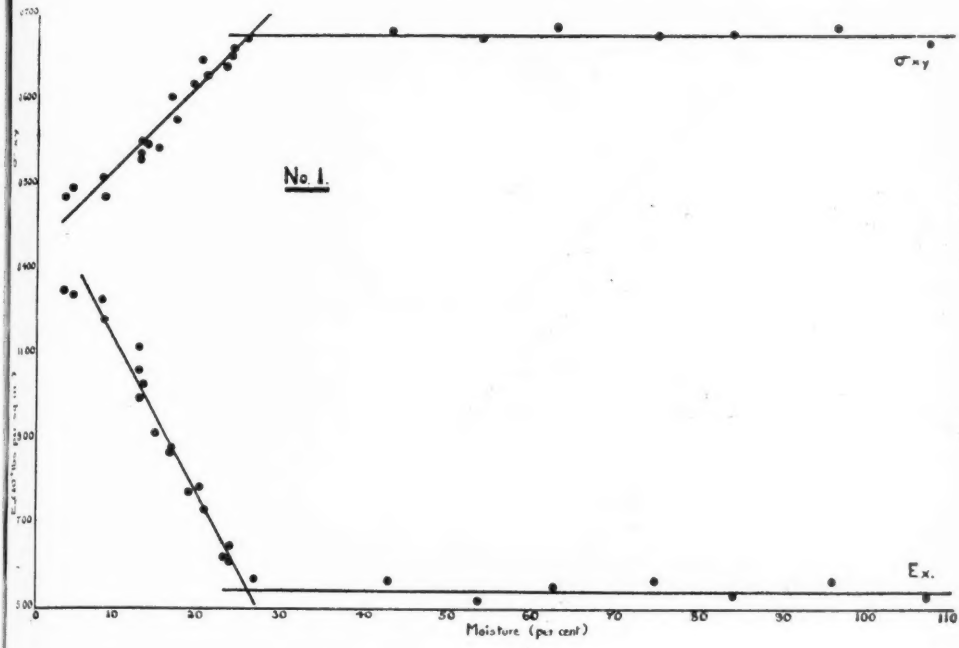
The results of the experiments can be seen by a glance at Figs. Nos. 1 to 6, where each result has been plotted against the corresponding moisture. The points in each case lie approximately on two straight lines which intersect in the region of 30 per cent. of moisture, this being the fibre saturation region of the wood. It will be noted that the constants vary considerably between about 0 and 30 per cent. of moisture, beyond which they remain nearly constant. All the values of Young's modulus, the modulus of rigidity and σ_{zx} decrease from about 0 to 30 per cent. of moisture whilst σ_{xy}, σ_{yx} and σ_{zy} increase within this region.

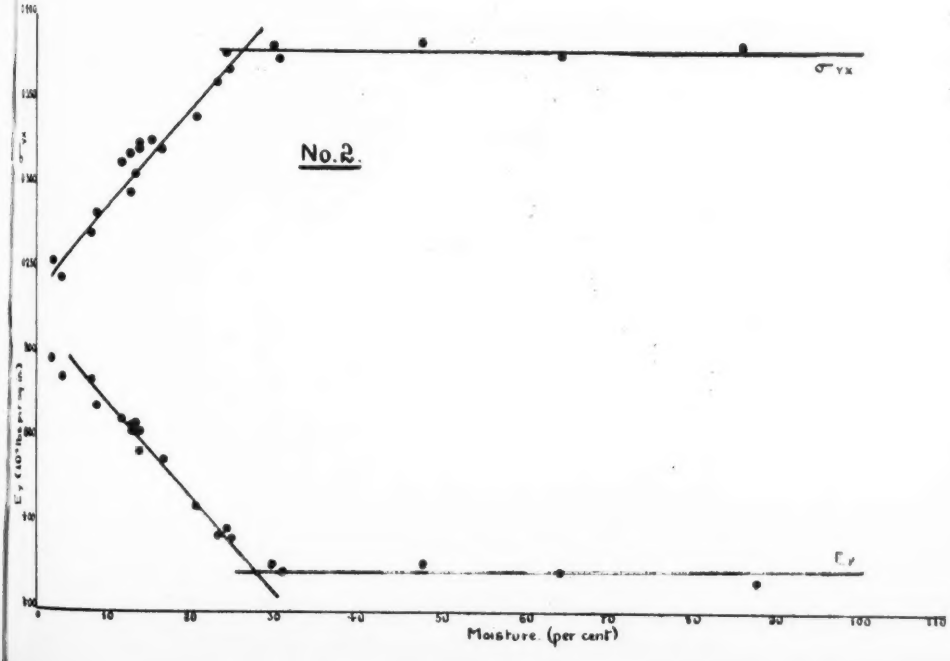
The relation between values of σ_{xz}, σ_{yz} and the percentage moisture are not included. These values are very small, being only about 0.02, and extra precautions during the experiments would have to be taken. Because of this and since the relations could be deduced from the other results the experiments were not undertaken. The relations can be obtained from the three symmetrical equalities

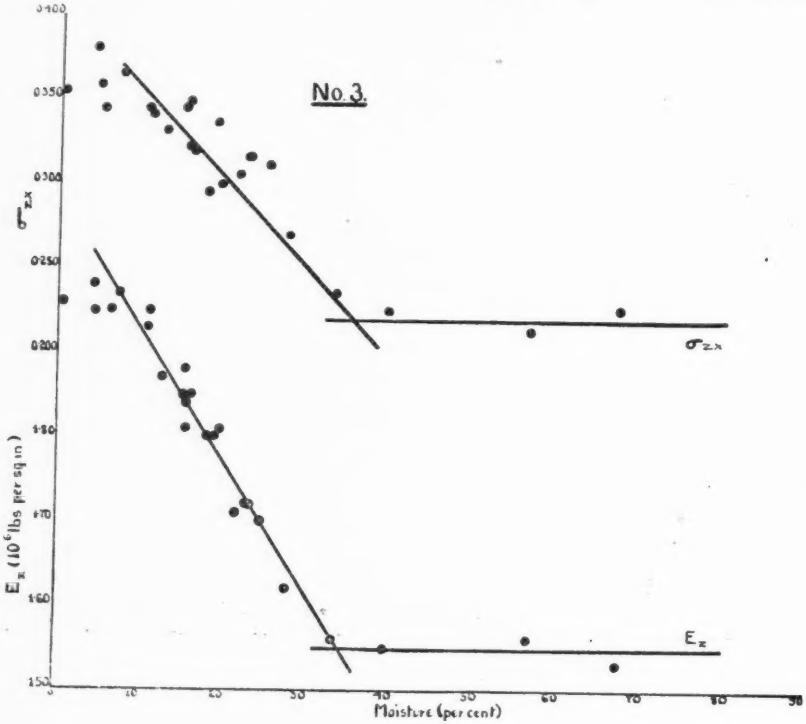
$$E_z/E_y = \sigma_{zy}/\sigma_{yz}, \qquad E_z/E_x = \sigma_{zx}/\sigma_{xz}, \qquad E_x/E_y = \sigma_{xy}/\sigma_{yx}.$$

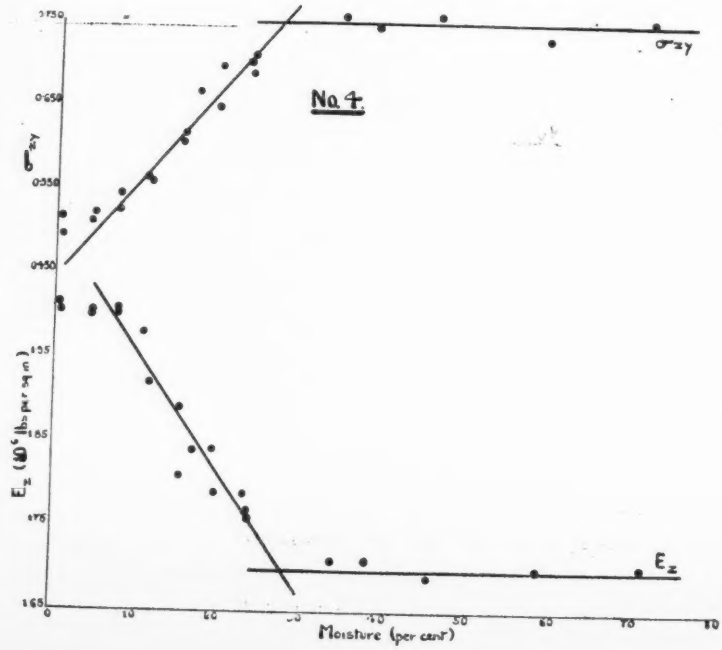
These equalities have been shown approximately to hold for spruce containing 12 per cent. of moisture and to be equal respectively to 700 S_y, 700 S_x and S_y/S_x* where S_y and S_x denote the shrinkages per unit length of dry timber in the directions YOY and XOX respectively, corresponding with 12 per cent. of moisture. In the case of the last of the equalities it will be found from a con-

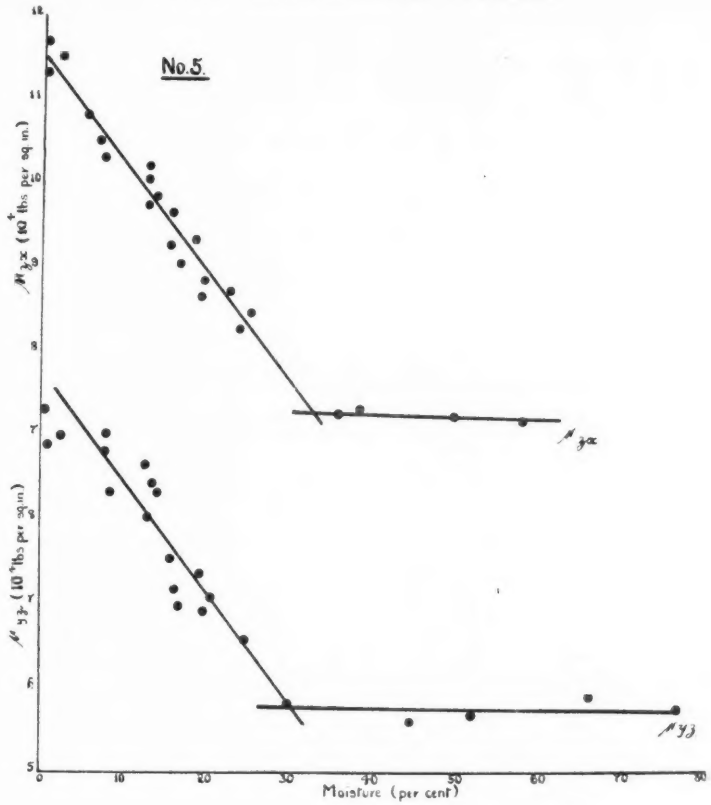
* Phil. Mag., May, 1922.

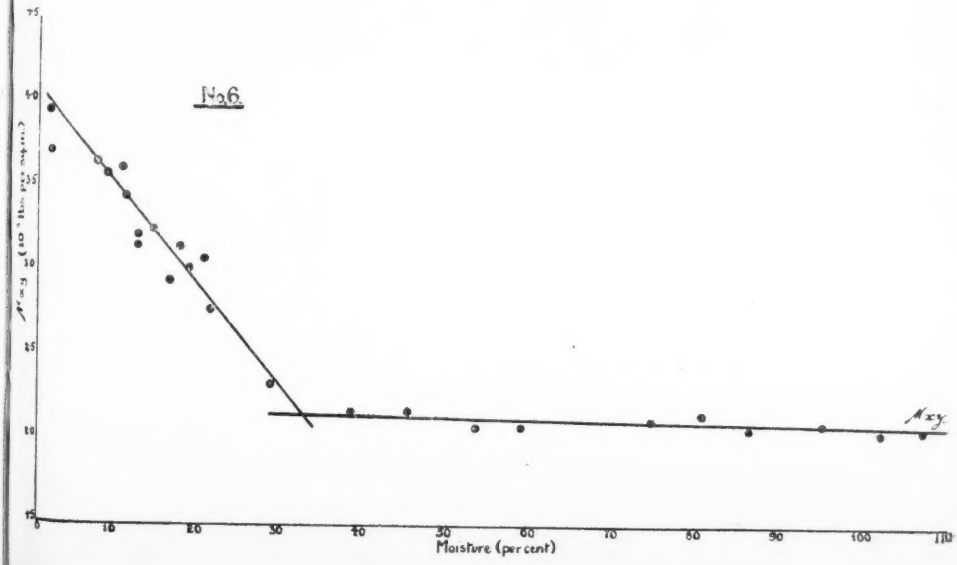

No. 1.
σ_{xy}
E_x.
Moisture (per cent)


No. 2.
σ_{yx}
E_y
Moisture (per cent)


No. 3.
σ_{zx}
σ_{zx}
E_z
E_z (10^6 lbs per sq in)
Moisture (per cent)

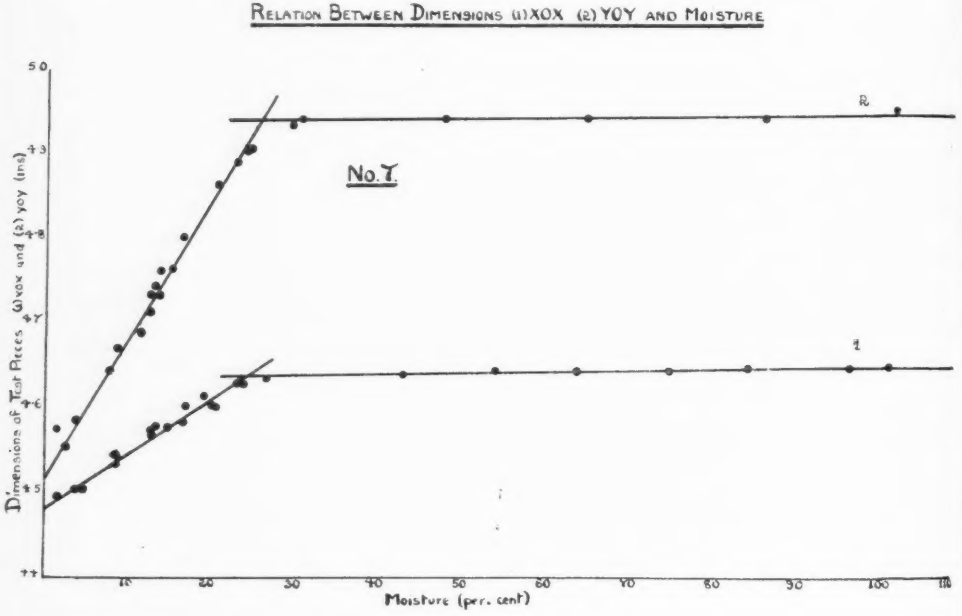

No. 4.
σ_{zy}
σ_{zy}
E_z
E_z (10^6 lbs per sq in)
Moisture (per cent)


No.5.
Moisture (per cent)


No6.
Moisture (per cent)

sideration of the curves on Figs. Nos. 1 and 2 that previous to the fibre saturation point $E_x/E_y = 1.63$ and $\sigma_{xy}/\sigma_{yx} = 1.73$ so that the ratios are approximately equal. Assuming the other two equalities to hold, the relations between σ_{yz} and σ_{xz} and the moistures must be similar to those between σ_{zy} and σ_{zx} shown on Figs. 3 and 4.

The relation between E_z and the moisture shown on Figs. 3 and 4 is very similar to that obtained by Tiemann. He, however, finds the relation, between zero moisture and the fibre saturation point, to be best represented by a slightly curved instead of a straight line. This may be due to the fact that in Tiemann's case each plotted value of the modulus corresponding with a particular humidity was the mean of a number of experiments on different pieces and hence the procedure differed from that adopted in this research.



Connection between Shrinkage and Moisture.

In the case of the test pieces corresponding with Figs. 1 and 2 the lengths which were in the directions XOX and YOY respectively were measured immediately after each weighing. The dimensions were then plotted against the corresponding moistures and resulted in the curves shown in Fig. 7. These are strikingly similar to the curves of Figs. 1 to 6 and show that the dimensions increase in a linear manner from dryness to the fibre saturation point and then remain constant. It follows that the relations between the shrinkages defined as (dimensions wet—dimensions dry/dimensions dry) and the moisture must be represented in a similar manner. It further follows that the relation between the shrinkage in volume and moisture must also be similar. For let S_x, S_y and S_v denote the shrinkages in the directions XOX, YOY and in the volume respectively per unit lengths and volumes of dry timber then

$$(1 + S_x)(1 + S_y) = 1 + S_v$$

therefore $S_x + S_y = S_v$ neglecting the small product $S_x S_y$.

The shrinkages in the direction *ZOZ* were very small, being less than 0.1 per cent. at and beyond the fibre saturation point and were accordingly neglected.

Effect of Continued Drying on Dimensions and Weight.

The shrinkages and moisture contents as previously defined are of little value unless the dimensions and weights of the pieces when dry attain to sensibly constant amounts. In order to test this, a number of pieces were prepared and placed in the stove, which was at a temperature of from 100 to 104 degs. C., and weighed and measured at intervals. The results in the case of the moisture are shown in Table 1, and it will be noted that in the case of the large block the moisture increases only slightly. In the case of the small block the increase is greater and the values are not so consistent. This block was however very small, and owing to convection currents and the rapidity with which it absorbed

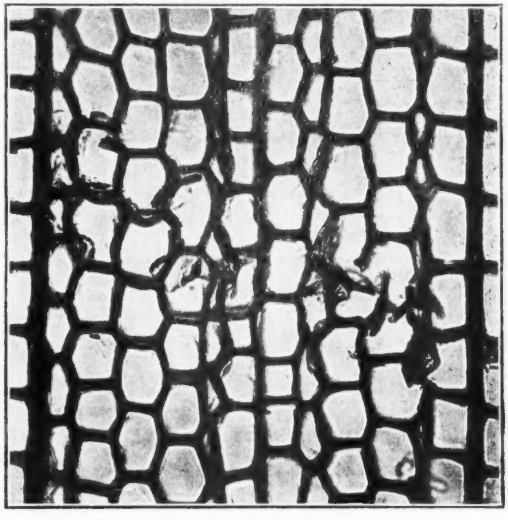


FIG. 8.

moisture from the atmosphere of the laboratory it was difficult to weigh accurately. The increase in the case of the chips and shavings is about the same as for the small block. When considering these increases it is worth noting that cellulose chars slightly at 100 degs. C., and the increase in the case of the small block, the chips and shavings over that of the large block may be due to the greater surface over volume and the consequent greater loss of weight owing to the charring of the proportionately greater surface.

The shrinkages are shown in Tables 2 and 3 and it will be noted that after a lengthy drying the increases are only slight.

From a consideration of the tables it seems reasonable to assume that after pieces of spruce have been dried at 100 degs. to 104 degs. C., for from 24 to 48 hours, they attain to practically constant weights and dimensions so that the shrinkages and moistures are definite properties of the material.

Damage to Cell Walls Due to Drying.

A consideration of some of the curves in Figs. 1 to 6 shows that some of the

points in the region of about 0 and 5 per cent. of moisture depart from the general linearity of the other points. It may at first appear that this is due to rupture of the cell walls during drying and a consequent weakening of the wood. If this had been so, however, the points would very probably have been more scattered than is the case, since each piece was dried at least twice. In order to test the effect of a severe drying on the cell walls a piece was dried at 110 degs. C. for four days and six sections were prepared for examination under the microscope. The cell walls were found to be damaged in the case of one only of the six sections and a photomicrograph of this section magnified about 150 diameters is shown in Fig. 8. Six sections were also prepared from a piece which had not been dried and no ruptures were evidenced in any one of these sections. A photomicrograph of one of them, also magnified about 150 diameters, is shown in

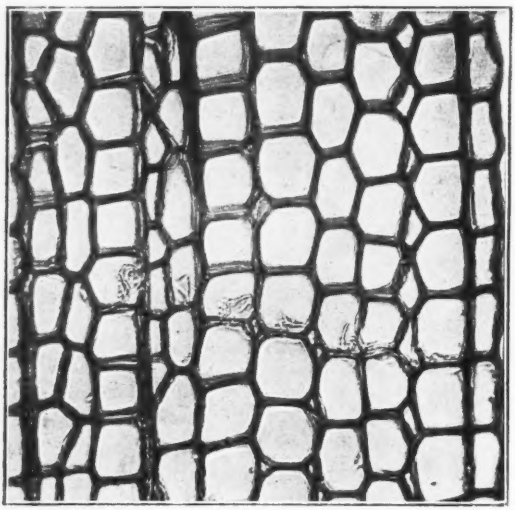


FIG. 9.

Fig. 9. It was therefore concluded that the departure from linearity between 0 and about 5 per cent. of moisture was not caused by damage to the cell walls due to drying.

Tiemann states that "no matter how dried, wood is found upon re-soaking to be weaker than it was originally." The pieces used by Tiemann were, however, generally larger and less homogeneous than those used in this research. If large saturated pieces are dried at 100 degs. C. or over damage to the cell walls will result and will usually be evidenced by the appearance of cracks. As the dimensions of the pieces are reduced and the drying is conducted more carefully a stage appears to be reached when no damage is evidenced. The pieces used in this research were small and the temperature of the stove was raised gradually over a period of a few hours. Pieces which had been wet were allowed to remain in the laboratory until the moisture fell to about 12 per cent. before they were placed in stove and if subsequent tests had to be performed no piece was allowed to remain in the stove for more than 48 hours at one time.

The writer wishes to thank Mr. H. A. Webb, M.A., for many suggestions and for the loan of Tiemann's paper.

TABLE 1.

Time in stove. hrs. mins.	Initial weights (grams). Block.	Block.	Chips.	Shavings.
	84.60	2.223	13.292	14.577
21.20	9.7	9.7	10.2	10.6
40.00	9.6	9.9	10.5	10.7
65.40	9.6	9.7	10.2	10.6
90.50	9.8	10.0	10.3	10.7
112.55	9.8	10.1	10.4	10.8
162.10	9.8	9.8	10.3	10.8
185.20	9.8	9.9	10.4	10.7
257.15	9.8	10.2	10.2	11.1
282.00	9.8	10.3	10.5	10.9
333.20	9.8	10.1	10.6	11.0
357.50	9.9	10.0	10.7	11.1
378.20	9.9	10.4	10.7	11.2
402.25	9.8	10.3	10.3	10.9
449.50	9.9	10.6	10.7	11.2
		Moisture (per cent.).		

TABLES 2 and 3.

Length of piece 7.022″.

Initial Density of piece 26.4 lbs./cu. ft.

Dimensions (ins.). *XOX.*	*YOY.*	S_{xx} 100.	S_{yx} 100.	Moisture per cent.	Time in stove. hrs. mins.
1.0162	0.2832	—	—	—	0.0
0.9965	0.2734	1.98	3.60	11.9	46.30
0.9964	0.2733	1.99	3.65	11.9	71.15
0.9965	0.2732	1.98	3.65	12.1	116.25
0.9963	0.2733	2.00	3.65	12.1	214.45
0.9963	0.2731	2.00	3.70	12.2	381.25

Length of piece 0.3022″.

Initial Density of piece 25.6 lbs./cu. ft.

Dimensions (ins.). *XOX.*	*YOY.*	100 S_x.	100 S_y.	Moisture per cent.	Time in stove. hrs. mins.
1.1945	1.5900	—	—	—	0.0
0.9906	1.5343	2.06	3.63	11.9	46.35
0.9898	1.5340	2.07	3.64	12.0	71.20
0.9882	1.5335	2.09	3.68	12.1	116.30
0.9879	1.5330	2.09	3.71	12.2	214.35
0.9872	1.5327	2.10	3.73	12.4	381.30

THE VIBRATION OF AIRSCREW BLADES.

BY CAPT. J. MORRIS, B.A., A.F.R.AE.S.

1. Airscrews are generally made of wood and owing to the relatively low Young's modulus of timber the blades of an airscrew have a large measure of flexibility. This flexibility is reflected in the comparatively low frequency of vibration of the blades with consequent undesirable effects. Firstly, there is a tendency for the blades to heat resulting in the laminations becoming unstuck, and secondly, there is considerable danger from resonance between the frequency of the blades and the firing impulses of the engine driving the airscrew. The result of this is heating and burning of the boss and ultimately failure of the airscrew. It may even result in failure of the airscrew shaft by shearing.

Airscrews are usually designed to meet certain aerodynamic needs. A certain amount of attention has been given to strength in the ordinary way, but in the opinion of the Author there has not been sufficient consideration of the oscillatory properties of the blades. Usually the question of vibration has been confined to the "flutter," that is to say, the fore and aft vibration of the blades. True, the blades are most flexible in this direction, but they are also flexible in the plane of rotation of the airscrew and it is extremely probable that many cases of failure could be traced to resonance between the frequency of vibration in the plane of rotation and the firing impulses of the engine. The resulting failure—especially if the airscrew shaft fails—might easily be attributed to a defect of the engine when actually it may be that the flexibility of the blades is entirely to blame.

The airscrew, as a rule, is one member of a more or less complex system. Whatever periods of vibration the blades of an airscrew have as a result, it can be shown that, in addition, the blades will have their ordinary periods in free vibration. When arranging for an airscrew for a particular engine the flexible properties of the blades should be known with a view to ascertaining whether these properties will give rise to undesirable effects. Actually the specifications for an airscrew should give a lower limit to the fundamental frequencies of the blades in both the fore and aft plane and the plane of rotation.

2. If a rod (not necessarily uniform in section), encastred at one end, be pulled aside and then let go, so as to oscillate in one plane, then the equation of motion can be shown to be

$$d^2 [EI_x (d^2y/dx^2)]/dx^2 = -\rho (A_x/g) (d^2y/dt^2) \quad . \quad . \quad (1)$$

where y is the displacement of an element dx at a distance x from the fixed end; A_x is the area of cross section at the element and I_x its moment of inertia for bending; ρ is the density of the material and E its Young's modulus.

If the rod is uniform in section equation (1) becomes

$$EId^4y/dx^4 = -\rho (A/g) (d^2y/dt^2) \quad . \quad . \quad . \quad . \quad (2)$$

This equation can be solved and from its solution we find that the fundamental frequency of vibration of the rod is very approximately the same as if the rod is regarded as light with a concentrated load of one-quarter the weight of the rod at its free end.

Actually the fundamental frequency in transverse vibration will be

$$(1/2\pi)\sqrt{(3EIg/Wl^3)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where W is $\frac{1}{4}$ the weight of the rod and l is its length.

The expression (3) can be written

$$(1/2\pi)\sqrt{(g/Wy_{11})} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$y_{11} = l^3/3EI$$

or the deflection due to unit load at the free end. If the rod is not uniform in section and y_{11} is the deflection due to unit load at the free end, there will be an equivalent tip load W, dependent on the shape of the rod, such that the fundamental frequency will still be given by the formula (4). There are well known experimental methods for finding both y_{11} and W.

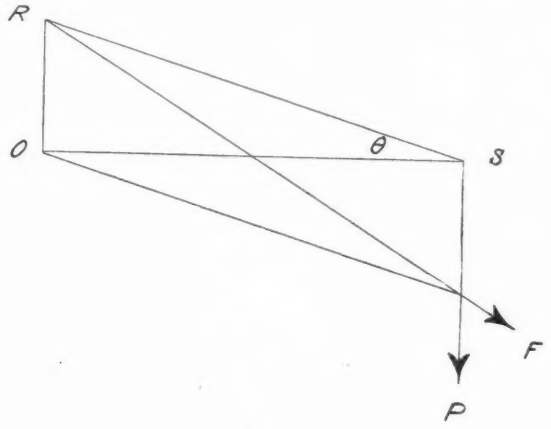


FIG. 1.

3. Airscrew blades, in addition to other forces, are subjected to centrifugal force. In order to investigate the effect of this force consider a light cantilever subjected to, at its free end, a load P at right angles to its normal line and a lateral load F (see Fig. 1).

Let OS be the normal line of the cantilever and let OR and PS be perpendicular to this line. Let RF be the line of action of F and SP the line of action of P; and let angle $RSO = \theta$.

If y_{11} and ϕ_{11} are the deflection and slope at the free end due to unit load alone, F being zero, it can be shown that the deflection and slope respectively when P and F both act as in Fig. 1 are

$$(P + F\theta)\,\gamma(a)\,y_{11} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$(P + F\theta)\,\Phi(a)\,\phi_{11} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where

$$\gamma(a) = 3(a \coth a - 1)/a^2 \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\Phi(a) = (\tanh \tfrac{1}{2}a)/\tfrac{1}{2}a \quad . \quad . \quad . \quad . \quad (8)$$

$$a \text{ being } \sqrt{(Fl^2/EI)} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

When a is small $\gamma(a)$ and $\Phi(a)$ are both practically unity and they do not differ appreciably from unity when a is as large as one radian.

4. Suppose F arises from the centrifugal force of a load W then

$$Fl^2/EI = Wl^3\Omega^2/EIg \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where Ω is the angular velocity of rotation.

Suppose (10) is small then $\gamma(a)$ and $\Phi(a)$ can each be taken as unity. Two special cases arise.

(1) The line of action of F passes through O (this will be the case for the

vibration of the airscrew blades in the plane of rotation) so that $\theta = 0$. Under these conditions the deflection and slope respectively become

$$Py_{11} \text{ and } P\varphi_{11}$$

so that the effect of the centrifugal force on the bending of the cantilever is negligible.

(2) The line of action of F is parallel to the normal line of the cantilever (this will be the case for vibration of the airscrew blades in a fore and aft plane).

In this case $\theta = -\gamma/a$ where γ is the deflection at the free end of the cantilever.

We find in this case

$$\gamma = Py_{11}/(1 + W\Omega^2 y_{11}/g) \quad . \quad . \quad . \quad . \quad (11)$$

and for the slope

$$\Phi = P\phi_{11}/(1 + W\Omega^2 y_{11}/g) \quad . \quad . \quad . \quad . \quad (12)$$

5. Consider next the frequency of the "flutter" or fore and aft oscillations of the airscrew blades.

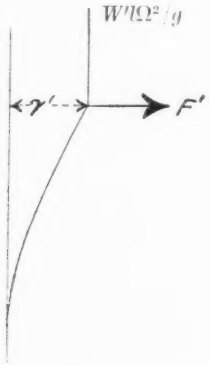


FIG. 2.

Let W' be the equivalent tip load and y'_{11} the equivalent coefficient of deflection. F' is the equivalent tip air force (see Fig. 2).

Having regard to the effect of the centrifugal force in this case, by the application of formula (11), we have for the equation of motion of W'

$$(1 + W'\Omega^2 y'_{11}/g)\,\gamma' = (F' - W'\ddot{\gamma}'/g)\,y'_{11} \quad . \quad . \quad . \quad (13)$$

or

$$[(W'y'_{11}/g)\,(D^2 + \Omega^2) + 1]\,\gamma' = F'y'_{11} \quad . \quad . \quad . \quad (14)$$

Thus the frequency of vibration is given by

$$(1/2\pi)\sqrt{(g/W'y'_{11} + \Omega^2)} \quad . \quad . \quad . \quad . \quad (15)$$

In this case the frequency of vibration is affected by the centrifugal force; the extent may be considerable and is dependent on the relative values of $g/W'y'_{11}$ and Ω^2.

6. Take next the motion in the plane of rotation of the airscrew (see Fig. 3).

Let θ be the angular displacement of the normal line of the airscrew blades and let γ_1 γ_2 be the displacements of conjugate elements of each of the two blades, each at a distance x from the centre of the airscrew.

We have for the equations of motion

$$d^2 (EI_x d^2\gamma_1/dx^2)/dx^2 = -F_1 - (\rho A_x/g)(d^2\gamma_1/dt^2 + xd^2\theta/dt^2) \quad . \quad (16)$$

and

$$d^2 (EI_x d^2\gamma_2/dx^2)/dx^2 = -F_2 - (\rho A_x/g)(d^2\gamma_2/dt^2 + xd^2\theta/dt^2) \quad . \quad (17)$$

where F_1 F_2 are the corresponding air loadings at the points, and the centrifugal forces have been neglected in accordance with case (1) in paragraph 4.

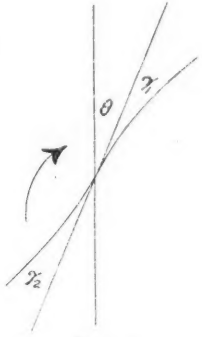


FIG. 3.

Thus

$$d^2 (EI_x d^2\gamma/dx^2)/dx^2 = -(\rho A_x/g)\, d^2\gamma/dt^2 \quad . \quad . \quad . \quad (18)$$

where

$$\gamma = \gamma_1 - \gamma_2$$

and

$$F_1 - F_2 = 0$$

But (18) is the equation for the free vibration of one blade regarded as a cantilever and this has been considered in paragraph 2.

Thus the blades will have, in addition to any other frequencies, their ordinary period for vibration in their plane of rotation.

In addition to giving rise to the frequencies considered, the flexibility of the blades in the plane of rotation may have an appreciable influence in lowering the speed of torsional resonance of the crankshaft, but this aspect is beyond the scope of the present investigation.

SCOTTISH BRANCH.

THIRD ANNUAL REPORT FOR YEAR ENDING 31st MAY, 1922.

The Executive Committee have pleasure in submitting the Third Annual Report of the activities of the Scottish branch of the Society.

A reduction in the number of members is no doubt due to the fact that aeronautical science is not receiving in this country the enthusiastic consideration which its importance deserves.

The following is the programme of lectures delivered during the Session 1921-22 :—

On 19th September a communication was read from Mr. Norman A. Yarrow, of Victoria, British Columbia, on "Civil Aviation in Canada." The Right Hon. Lord Weir of Eastwood presided.

On 17th October a lecture was delivered by Col. V. C. Richmond, of the Airship Dept. of the Directorate of Research, on "The Organisation of a Colonial Airship Service." Mr. A. J. Campbell presided.

From 3rd to 8th October a series of lectures was given by Sir Ross and Sir Keith Smith in Hengler's Circus, on their Flight to Australia, illustrated by kinema views. The chair was occupied on six successive evenings by the following :—Monday, the Right Hon. the Lord Provost; Tuesday, the Right Hon. Lord Weir of Eastwood; Wednesday, Sir W. F. Russell, Chairman of the Chamber of Commerce; Thursday, Mr. W. Gillies, LL.D., Pres. of the Royal Philosophical Society; Friday, Mr. Harold Yarrow, Chairman of the Institute of Engineers and Shipbuilders. These lectures were attended by audiences ranging from 800 on 3rd October to 1,300 on Saturday evening, 8th October.

On 31st October a lecture was delivered in the Natural Philosophy Class-room of the University by Professor Gordon Gray, on "Research Work and the Application of Gyroscopes to Aviation." Mr. James G. Weir, C.M.G., presided and there was an audience of nearly 200.

On Monday, 14th November, a lecture was delivered in the Engineers' Institute by Air-Marshal Sir Hugh Trenchard, Bart., C.B., K.C.B., Chief of the Air Staff, on "Aids to an Auxiliary Air Force." The Right Hon. Lord Weir of Eastwood presided.

On Wednesday, 7th December, a meeting of the ex-Airmen and Students' Section was held in the University when a paper was read by Mr. C. R. Catesby, on "Flying in 1921."

On Monday, 12th December, 1921, a lecture was delivered in the Technical College by Col. Gold, of the Meteorological Dept. of the Air Ministry, on "The Application of Meteorology to Aviation." The lecture was illustrated by lantern slides and Professor Mellanby presided.

On 25th January a lecture was given in the University to the ex-Airmen's and Students' Section by Major Cleghorn, B.Sc., on "Aeroplane Repairs During War Time."

On 27th February a lecture was delivered in the Technical College by Mr. Alan E. L. Chorlton, C.B.E., M.Inst.C.E., M.I.Mech.E., on "Special Light Weight Engines." There was an attendance of over 300 and Professor Mellanby presided.

On Monday, 10th April, the annual lecture to the Cadets of the Public Schools of Glasgow was delivered by Major D. C. M. Hume, of the Seaplane Research Dept. of the Air Ministry, in the large hall of the Engineers' Institute, on "Boats that Fly." The hall was completely filled by the Cadets in uniform of the leading Public Schools, the audience numbering over 300. The lecture was illustrated by limelight views. Major Cleghorn presided and a cordial vote of thanks was accorded to the lecturer on the motion of Mr. Harold E. Yarrow.

Work Among the Students.

This most important work has proceeded in a satisfactory manner during the past session. On Friday, 21st October, the Hon. Secretary addressed Professor Mellanby's third year class, the number present being 80, of whom 20 filled up the form for student membership.

On 24th October Professor Mellanby's second year class numbered 130, of whom 17 filled up forms for student membership.

On 26th October Professor Cormack's third and fourth year classes were addressed, at each of which 180 were present. The combined adhesions from both classes was 71.

On 27th October Professor Andrew Gray's Natural Philosophy Class was addressed, at which 200 were present, and the number of adhesions was 21.

On the same date, at a later hour, Professor Gordon Gray's Nos. 1 and 2 classes were addressed, at each of which 200 were present. The number of adhesions from both classes was 85.

In this way over 1,000 engineering students were addressed and the literature of the Society was forwarded to those signifying their special interest, numbering in all 214.

This work among the students the executive regards as of the utmost importance and they consider it as a work which will rapidly develop.

In regard to finance the position is as follows:—The annual subscriptions received amount to £222 2s. 8d., compared with £231 1s. 0d. last year. There have been no grants from the Universities and therefore a diminished amount has been paid for lectures. The ordinary expenses, owing to this and other causes, have dropped from £485 3s. 5d. to £340 18s. 0d. The balance now in hand from the Initial Establishment Account is about £400. This amount it is intended to spread over at least two years, and before it is finally expended it is hoped that aeronautical science will have come to its own, and that the number of new members joining the Society will be sufficient to make the income meet the moderate expense which is needed to keep the Society in that state of vigour which is so important for the work in Scotland in view of the large number of engineering students in the University, with whom the Society is keeping in close touch at considerable cost.

THE ROYAL AERONAUTICAL

Abstract of Cash Receipts and Payments

RECEIPTS.

	£ s. d.	£ s. d.
Cash in Bank on Temporary Loan less due to Hon. Secretary at 1st June, 1921		529 10 6
ORDINARY.		
Annual Subscriptions received from 1st June, 1921	222 2 8	
Refund of Income Tax	3 16 0	
		225 18 8
EXTRAORDINARY.		
Special Donations	10 10 0	
Special Donation to Initial Establishment Fund ...	5 0 0	
Interest on Corporation Mortgage less Tax	15 16 2	
Part Proceeds from Ross-Smith Lectures	13 1 8	
		44 7 10
		£799 17 6

GLASGOW, *7th July, 1922*

SOCIETY (SCOTTISH BRANCH).

for the year ended 31st May, 1922.

EXPENDITURE.

	£ s. d.	£ s. d.
Amount paid to Head Office, London, being one half of Annual Subscriptions received by this Branch from 1st June, 1921, to 16th Dec., 1921, as per arrangement	45 11 0	
Miscellaneous Expenses *re* Lectures, less Contributions received	58 19 0	
Printing	4 10 8	
Typewriting Supplies, Stationery, etc.	14 13 10	
Postages, Telegrams and Trunk Calls	20 2 0	
Advertising	3 1 0	
HON. SECRETARY'S EXPENSES—		
(1) Travelling Expenses £3 3 0		
(2) Clerical Assistance 85 2 6		
(3) Allowance for use of Office and Staff, Typewriters and Sundry Expenses from 1st June, 1921, to 31st May, 1922 ... ... 105 15 0		
	194 0 6	
		340 18 0
Balance of Funds on hand made up as follows:—		
Cash in Bank	33 13 6	
On Deposit Receipt	100 0 0	
On Loan with Glasgow Corporation	300 0 0	
In hands of Hon. Secretary	25 5 6	
		458 19 0
		£799 17 0

Audited and found correct.

(Sgd.) J. WYLLIE GUILD & BALLANTINE, C.A., *Auditors.*

PROCEEDINGS.

FOURTH MEETING, 59th SESSION.

An Ordinary General Meeting of the Society was held at the Royal Society of Arts, John Street, Adelphi, London, on Thursday, November 16th, 1922, the Chairman, Professor L. Bairstow, in the chair, when a lecture was delivered by Mr. R. McKinnon Wood on "The Co-relation of Model and Full Scale Work."

The CHAIRMAN, in calling upon Mr. McKinnon Wood to read his paper, said that Mr. Wood was in a particularly advantageous position to deal with this subject, since for very many years he had been in charge of full scale experimental work in aerodynamics at the Royal Aircraft Establishment and, in addition, had had at his disposal wind channels for the carrying out of model work. The greater part of the model work in this country was done at the National Physical Laboratory, and with the members of the staff of the National Physical Laboratory Mr. Wood was on intimate terms. They visited each other's establishments and occasionally exchanged members, so that both sides of the subject, the full scale and the model work, were known to Mr. Wood at first hand. In the past the subject had been an extremely controversial one, one of the controversialists being himself, but he was glad to say that in many respects the controversy had now gone out of the subject, although that did not in any sense mean that the interest had gone out of it. They were realising that in very many ways new problems were turning up. The simpler theories with which they began aeronautics were gradually giving way to more complex ones, and the rules to which they had tied their faith in the early days of aeronautics, and which were considered complete, were now known to be somewhat incomplete. That was a tribute in itself to those people who were working on aeronautics. They were willing to see new moves and to develop gradually from main principles without in any sense stating that those main principles were to be thrown overboard. He then asked Mr. McKinnon Wood to give them an account of the present position of and knowledge as to the relation between model and full scale experiments.

Mr. MCKINNON WOOD then read his paper.

THE CO-RELATION OF MODEL AND FULL SCALE WORK.

Introductory.

Model tests are capable of a wide usefulness in aeronautical engineering if the model work can be satisfactorily co-related with the full scale; and they have, in fact, played a very important part. They provide a rapid and economic method of testing new ideas and investigating aerodynamic theories; of discovering the best form for wing sections, fuselages, airship hulls, etc., and of guiding the designer of aircraft in obtaining the best proportions and distribution of the component parts of the craft in order to attain speed, climb, stability, and ease of control. Many designing firms make use of wind channels, and work is also carried out at the National Physical Laboratory and the Royal Aircraft Establishment directly to aid designers. I believe that a fuller use of wind channels for design purposes could be made with advantage and that their value will be greatly extended when the testing of models with the airscrew running has become a matter of routine. This is not prohibitively difficult or costly. The action of the airscrew is important in its effects upon trim and stability and in producing

a tendency to turn, effects which one may guess but cannot calculate with any certainty.

May I give two illustrations of the value of the model in design? In one case in my experience the climb of a new aeroplane was found to be considerably poorer than was expected in view of the power, weight, and wing area, and wind-channel tests carried out after the aeroplane had been built and flown showed that the poor performance could have been predicted. In another case an aeroplane in service acquired considerable unpopularity on account of its tendency to turn when the engine was opened out. This could have been predicted from a wind channel test, in this case of a somewhat elaborate nature.

We cannot calculate the lift or drag of a wing or the centre of pressure or the angle of downwash behind it, the thrust or torque of an airscrew, the drag of fuselage or other parts, the mutual interaction of airscrew and fuselage. The only data upon which we can base design is that derived from model or full scale experiments. We can piece such data together and so build up the complete aircraft, as we calculate the thrust and torque of an airscrew from aerofoil data or the drag of a complete aeroplane from component parts; but the validity of such a calculation requires experimental confirmation. Our experimental data can be acquired much more rapidly, safely and economically by model tests than by full scale work. The subject of this paper is the "if" of my first sentence: If model work can be satisfactorily ro-related with the full scale. Unless we can establish some relation between the model work and the full scale, our model work must be of very uncertain value. We may endeavour to do this theoretically or experimentally, and I shall start with the theoretical aspect of the question.

Theory.

Aircraft moves in a medium possessing the following physical properties:—

(1) Fluidity—the property that, when the fluid is at rest, the stress over an elemental surface is normal to it.

(2) Viscosity—the property of resistance to a shearing motion, which introduces tangential stress when part of the fluid is in motion relative to a neighbouring part.

(3) Compressibility—the density varies, directly as the pressure and inversely as the temperature.

Excluding very fast-moving bodies such as shells, bombs and airscrew blades, we may regard air as incompressible, as the variations of pressure produced by bodies moving at the speed of aircraft are not large enough to cause important changes of density.

The important properties in most aeronautical work are the fluidity and the viscosity, and it will be shown later that, although viscosity is an essential property in all aerodynamic problems, these problems may frequently be divided into two parts, in one of which it is reasonable to suppose that viscosity plays an insignificant part.

The branch of mathematics known as hydrodynamics deals chiefly with fluids possessing only the first property—fluidity. In such a fluid the relation between model and full scale would be simple and definite. The pressure at any point on a body would be proportional to:—

(1) The density of the fluid (ρ).

(2) The square of the speed of motion through the fluid (V) or in mathematical symbols

$$p = k\rho v^2 \text{ where } k \text{ is a constant.}$$

In any actual fluid the viscosity must enter into this equation. Experiment

has shown that the viscous couples are proportional to the rate of shearing and a consideration of the dimensions of the coefficient of viscosity shows that it must enter into the equation :—

$$p = k\rho v^2$$

in the form

$$k = f(vl\rho/\mu)$$

where μ is the coefficient of viscosity and l a linear dimension of the model.

The equation expresses the Law of Dynamic Similarity for a viscous incompressible fluid. Expressed in words, it states that the pressure at corresponding points full scale and model will be in the ratio of the corresponding values of ρv^2 if the value of $vl\rho/\mu$ is the same; but provides no indication of the manner in which the pressure may vary if this quantity is different.

In order to conform to this law we should require to test a one-tenth scale model at ten times the full scale speed. This would require a very great expenditure of power and necessitate measuring very large forces; but apart from this, the results would be vitiated by the air ceasing to be virtually incompressible under the high pressures which would occur at such speeds.

This Law of Dynamic Similarity cannot therefore be conformed to in the wind channel as we use it. But there is another possible method by which models might be tested at the full scale value of $vl\rho/\mu$. If a model to a scale of 1/10 be tested in a channel built inside a strong steel chamber filled with air at 10 atmospheres pressure at atmospheric temperature, the value of $vl\rho/\mu$ is the same as that of the full scale at the same speed. The forces to be measured will be ten times as great per unit area as on the full scale, and this introduces difficulties in supporting the model which would make it desirable to work at lower velocities with a compensating increase in density and so reduce the forces to convenient values. I believe that the not inconsiderable difficulties which would arise in constructing a high pressure channel and designing suitable balances can be overcome. The cost of this method of experimenting would be greater than that of an atmospheric channel, but still much less than that of full scale work. The high pressure channel, however, belongs to the future, and my concern is here with the use of present apparatus.

Variation of the coefficient k in the equation

$$p = k\rho v^2$$

as we vary the size of the model has been called a "scale effect," and we have seen that the same variation should be produced by increasing the scale, the speed or the density of the fluid or by decreasing the viscosity in the same proportion.

Discussion of the nature of viscous flow may conveniently be started by considering the flow past a body consisting only of a surface moving in its own plane. The layer of fluid in immediate contact with the solid surface is agreed to be at rest relative to the surface, and the velocity builds up as we pass outwards from it. On the assumption that the flow is steady laminar flow the velocity gradient and the drag exerted on the surface can be calculated. The drag is proportional to the speed. Experiments with fluids passing down pipes of small bore at low speeds have given the calculated value of the resistance, but at higher speeds it was found that the resistance increased more nearly as the square of the speeds. It was also shown by introducing a filament of coloured liquid into water flowing along a glass tube that the flow was steady over the range of speeds for which the resistance varied as the speed, and became turbulent when the law of resistance changed. The state of steady flow occurs, however, only at very low values of $vl\rho/\mu$, and the values with which we are concerned in aeronautics are well inside the range of turbulent flow.

The resistance of thin smooth flat plates edge-on to the wind has been found by experiment to follow the law

$$k = (vl\rho/\mu)^{-0.15}$$

over a wide range.

We are generally concerned with bodies of some thickness whose drag is composed of two parts :—

(1) The resultant of the pressures normal to its surface.
(2) The resultant of the pressures tangential to its surface.

The distribution of pressure over bodies such as airship hulls can be calculated for the case of an inviscid fluid, and the results have been compared in some cases with measurements made upon a model. The agreement of calculated and measured values is very close over the nose, but they differ widely at the tail. The calculated values have no resultant, but the measured values integrate to give a drag force. The effect of viscosity is therefore two-fold :—

(1) It modifies the form of the flow so that the normal pressures have a resultant.

(2) Tangential forces are exerted on the body.

The resultant drag is composed of these two parts in proportions depending upon the shape of the body. Photographs of the flow round fair-shaped bodies indicate that the streamlines do not follow the body to the extreme tail, but break away from the body at a greater or less distance, according to the fineness of the body and enclose a "dead water" region. The pressure therefore fails to build up to the "pitot" value, $\frac{1}{2}\rho v^2$, which occurs at both ends in the calculated values for non-viscous flow, and a resultant force is produced. The part of the drag due to the normal pressures is commonly greater than that due to the tangential forces, and it is evident that scale effect upon drag is not confined to the component due to the tangential forces.

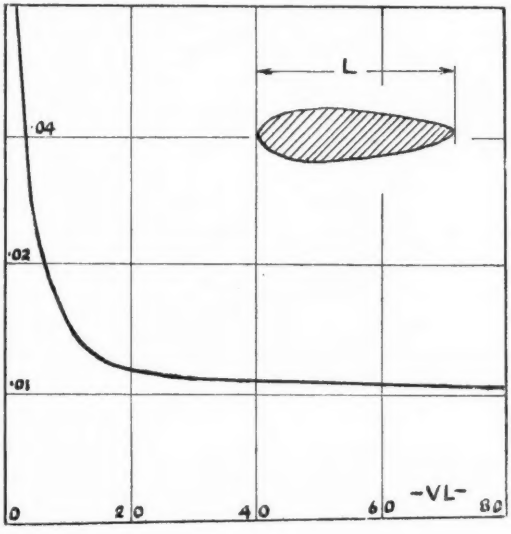


FIG. 1.

Drag per unit length of an aeroplane strut divided by ρLV^2 showing the variation with $VL\rho/\mu$.

I reproduce a curve obtained in a wind channel showing the scale effect upon the drag of a fine shaped body (an aeroplane strut). The scale effect is large at the lower values of $vl\rho/\mu$, but is quite small at the high values. This particular case is typical of results obtained with similar forms of bodies and we conclude

that, while experiments upon small models at low speeds may be of little value, it may be possible to carry out model tests at sufficiently high values of vl to give a close approximation to the full scale.

So far we have considered only symmetrical bodies moving along their axes or planes of symmetry. A body which is not symmetrical with respect to the direction of its motion experiences both a drag and a cross wind force or lift. The mean pressure of the air is less on one side than on the other and the velocity is greater. The principle of conservation of energy leads to Bernoulli's equation connecting pressure and velocity:—

$$p + \tfrac{1}{2}\rho v^2 = \text{const.}$$

except to the extent to which energy is degraded into eddies and heat by the action of the viscous forces. This degradation of energy is small if the drag is small in comparison with the lift, and Bernoulli's equation is closely true for the streamlines which do not pass very close to the surface of a wing. In the language of hydrodynamics, the flow is composed of a translation (or linear motion) and a circulation round the wing. It can be shown that the existence of tangential stress in the fluid is essential for the production by a wing of a circulation round it. Lift is therefore dependent, as drag is dependent, upon the action of viscosity.

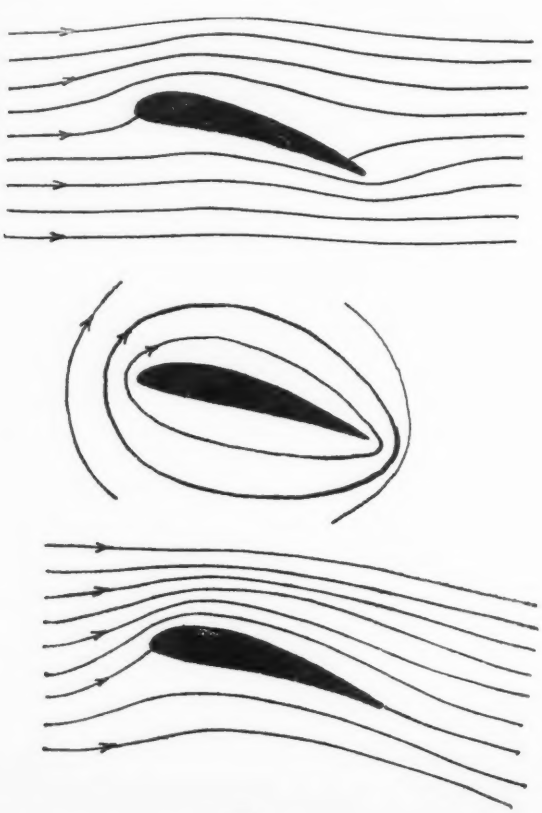


FIG. 2.

Diagram 1 shows the form of flow which should theoretically exist past a wing moving in a fluid having no viscosity, the wing being supposed to extend indefinitely in either direction across the stream so that the flow is two-dimensional. Diagram 2 shows the fluid circulating round a wing. In both cases the form of the streamlines is quite definite. We may conceive these two motions to be superimposed, and so obtain a series of flow diagrams of the type shown in Diagram 3. These are all possible forms of non-viscous flow, although the circulation could not be created by the motion of the wing without the action of viscosity. The exact form of flow depends upon the relative strengths of the linear and the cyclic motions.

It seems that a good approximation to the lift of a wing might be obtained by a calculation by non-viscous theory if we could determine what governs the strength of the circulation which is produced through viscosity; and in relation to scale effect the question of interest is whether this depends upon the magnitude of the viscosity or only upon the existence of viscosity. In the latter case we have a reason for expecting no scale effect upon lift.

It will be observed that the flow pattern of Diagram 1 presents difficulties when regarded as a possible form of flow in a real gas. Firstly, at the edges the velocity is theoretically infinite, which violates the assumption of constant density as air must expand greatly at this point; secondly streamlines very close to the wing double back twice upon themselves and viscosity would bring large couples into play to resist this. This form of flow would at once break down by the formation of eddies, and Joukowski has endeavoured to calculate lift on the assumption that the circulation set up is just sufficient to avoid this form of flow by bringing the "stagnation point" to the sharp edge. Difficulties arise because the circulation which secures this condition at the trailing edge is not in general the same as that which is required to secure it at the leading edge and the method also breaks down if neither leading nor trailing edge is at all sharp. These troubles were avoided by Joukowski by experimenting with aerofoil sections with well-rounded leading edges and sharp trailing edges. The illustration showing some results of his work indicates, I think, that his method gave

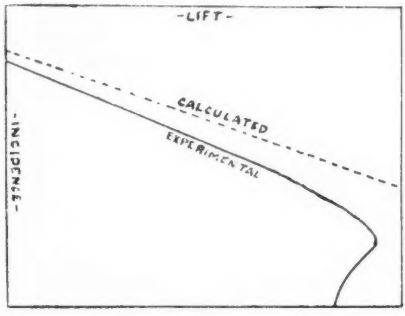


FIG. 3.

Comparison of Joukowsky's calculations of lift with measured values.

a remarkably good approximation to the facts. It does not, of course, provide any explanation of the breakdown of lift at the critical angle. It is not improbable that the actual flow is to some extent periodic, and it is, I think, certain that the streamlines do not completely come together at the trailing edge; under the wing the stream probably leaves the aerofoil at the trailing edge, while above it leaves at a point, near the trailing edge of fine angles of incidence, moving forward slowly at first and rapidly as the critical angle is approached. There is always a small turbulent wake enclosed between the convergent streams, which

becomes large when the wing stalls. The circulation and the lift would therefore be less than this method estimates. The interest of this theory in relation to our subject lies in the implication that the mechanism of lift depends principally upon the existence of viscosity and little upon its magnitude, leading us to expect little scale effect upon lift. We should expect larger scale effect where the edges of the aerofoil were well rounded.

Prandtl and some others working with him have also applied the theory of non-viscous fluids to aeronautics, but their work rests upon a less conjectural basis. We have so far considered only two-dimensional flow, to which we can only approximate in practice by the use of aerofoils of high aspect ratio. Starting from the two-dimensional viscous flow round a wing of given section and of infinite aspect ratio, Prandtl has deduced valuable theorems about the three-dimensional flow round a wing of finite aspect ratio, so that from a test of a wing of any one aspect ratio the characteristics of wings of the same section and of other aspect ratios may be calculated. Other theorems deal with multi-plane systems, with the influence of the proximity of the ground and of the walls of a wind channel.

The layer of fluid in contact with a body is at rest relative to the body, and the velocity gradient close to the body is steep in comparison with gradients elsewhere in the fluid. Measurements of velocity in air flowing along a surface show the very small distance at which there is a sensible reduction of velocity in comparison with the length of surface traversed. The tangential stresses in the fluid due to viscosity are therefore relatively great close to the surface of the body, and while viscosity may be responsible for the lift by producing a circulation, the flow at a little distance from the wing may be calculable by arguments which ignore the viscosity if they include the circulation to which it gives rise. We may summarise the argument by saying that the action of viscosity is concentrated towards the surface of the body. This constitutes the justification of Prandtl's application of non-viscous hydrodynamics to aeronautics. Another well-known application of non-viscous hydrodynamics is the theorem of Froude relating to screw propulsion, which has been subsequently combined with the Aerofoil Theory of Airscrews, most successfully, I think, by H. Glauert's recent development of Prandtl's work

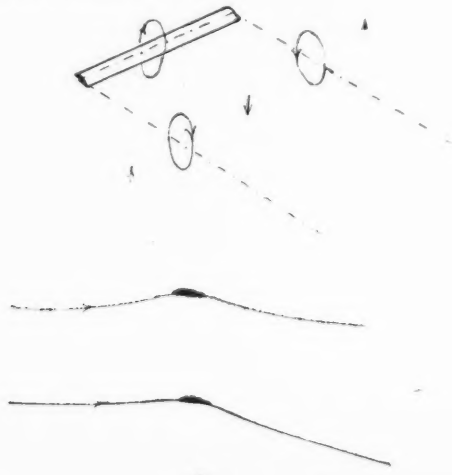


FIG. 4.

I can only give here a brief outline of Prandtl's work. Consider a finite wing experiencing a uniform lift along its span. There will be a uniform circula-

tion along the span which cannot cease at the tip, as by a theorem of hydrodynamics vortices cannot have a free end. A vortex must therefore trail from each tip. (In practice the lift will not be uniform to the tips, and the wing will therefore trail a sheet of vortices, the strength of the vortices from each portion corresponding to the fall of lift.) Comparing the finite wing with a hypothetical wing of infinite span, it will be seen that the trailing vortex system gives a downwash trend to the stream within the span and an upward trend beyond the tips. Whereas the flow before and behind the infinite wing is symmetrical (on the basis of non-viscous flow with circulation), the downwash behind the finite wing is greater than the upwash ahead, and the air at the wing has a downward trend. The wing moves in a downwash of its own producing.

This downwash at the wing can be easily calculated with sufficient accuracy. The method of estimating the lift and drag of the finite wing from those of the infinite wing is shown by the table in which ϵ represents the downwash at the wing :—

Aspect ratio.	Infinite.	Finite.
Lift coefficient	k_L	k_L
Incidence	α	$\alpha + \epsilon$
Drag coefficient	k_D	$k_D + \epsilon k_L$

This shows the incidence at which the two wings give the same lift and the drag coefficients which then obtain.

Infinite aspect ratio is the datum of the theory, but, of course, in practice the datum will be a test upon a wing of some finite aspect ratio.

Prandtl calls this increment of the drag at given lift by the trailing vortex system the "induced drag," and the drag of the wing of infinite aspect ratio and of the same section he calls the "profile drag." If there is no scale effect on lift, we may expect scale effect on drag to be confined to the "profile drag," and the variation to be greater in proportion to the whole drag at the smaller angles of incidence.

These circulation theorems have received considerable experimental verification. They are found to give with good accuracy the effects of change of aspect ratio and the direction of flow at points not too close to the wing. Unfortunately, the most useful calculation of direction of flow, the downwash at the usual position for the tailplane, is complicated by the rolling up of the vortex sheet, and has not so far been accurately made.

In so far as reliance might be placed upon this form of calculation, it is clear that a solution obtained by a model experiment should be independent of scale, and a class of problems exist in which wind channel results apply without doubt to the full scale. As an illustration there should be no scale effect upon the relation between lift and downwash, whatever scale effect may exist between lift, drag and incidence.

To summarise the conclusions of the theoretical discussion :—

(1) There is a law of dynamic similarity to which we do not conform in our model experiments.

(2) Until the mathematical theory of the flow of a viscous fluid is further advanced we cannot arrive at a precise or certain theory of scale effect when this law is violated.

(3) Our experience leads us to believe that by experimenting with models to the larger scales which we use we obtain a good approximation to the full scale conditions.

(4) We have some reason to believe that scale effect upon lift will be less than upon drag (until the stalling angle is approached).

(5) Certain applications of non-viscous theory to aeronautics are justifiable, and these are independent of scale.

Conditions of Model Tests.

In practice the conditions under which model work is conducted differ from those of full scale flight, and the co-relation of model and full scale involves the co-relation of these conditions.

Model experiments may take various forms. We may test a model of a complete craft in free flight or we may make measurements upon the whole or upon parts of a model aircraft in a constrained motion. In the former case, generally employed for investigating or demonstrating stability of flight, we must conform to certain conditions, which do not concern us in the latter case. The density of the model must be the same in proportion to the density of the air, the mass must be similarly distributed, and the action of gravity introduces the condition that v^2/lg must be the same. These conditions are easily obtained.

Experiments in constrained motion are employed for measurement of air forces, and I am confining my attention to this, the larger side of model work. In the case of rectilinear motion at constant speed the forces depend upon the relative motion, and it is convenient for a variety of reasons to hold the model fixed and cause the air to move past it. The apparatus employed is the wind channel or tunnel, of which two distinct types are in use. In one, the stream of air is an open jet, but in those used in this country the stream is bounded by four flat walls. To represent rotation the model must be rotated, and no difficulty arises in representing an airscrew rotating or a wing banking. The yawing and pitching motions do, however, present difficulties, as they involve in a wind channel change of the yaw or incidence, which they do not necessarily involve in flight. Producing the rotation without change of attitude requires a curved flight path, and the model must be moved through the air to secure strictly the correct conditions; but it is usual to obtain the forces required by causing the model to execute a small oscillation in the wind channel, assuming that the damping of the oscillation is entirely due to the angular velocity. I have, however, only space here to discuss the case of straight uniform motion.

The wind channel is designed to produce a steady parallel flow of uniform velocity, and the variations of speed observed in a good channel are of the order of 1 per cent. There is a very small drop of pressure as we pass downstream, for which allowance is made when necessary, as in testing airship models. The air is drawn in through a honeycomb to avoid large eddies, but it must be full of small eddies, and it is possible that this micro-turbulence may have some influence upon the experiments, although I do not believe that it introduces serious errors.

The method adopted for the support of the model in the channel is of great importance. The introduction of clumsy parts into the channel is a fruitful source of trouble. The balances are situate outside the stream, and in the best practice the model is suspended from them as far as possible by fine wires. Even these offer a considerable resistance and must be reduced to a minimum. Correction is easily made for that part of the drag of supports which acts directly upon the model; but it is also important to ensure that the supports do not cause a serious deflection of the flow.

The walls of the channel exercise a constraint upon the flow, and influence the results to an extent which depends upon the relative size of the model and the magnitude of the air forces. The Froude propulsion theorems have been extended to provide a theory for correcting tests of airscrews in a channel, and the work of Prandtl provides a theory for correcting tests of wings. The latter deserves some discussion here.

If the walls of the channel were optical mirrors, series of images of the model would be seen. The model in the channel is more or less aerodynamically equivalent to a model flying in formation with these images. By the circulation

aerofoil theory the influence of these images on the model may be calculated. The corrections are :—

$$\Delta\alpha = 16\,Sk_L/h^2$$
$$\Delta k_D = 0.27\,Sk_L^2/h^2$$

where S is lifting area of model and h the breadth of a square channel.

The following table shows the percentage correction applicable to the lift/drag ratio of biplane wings 6in. × 36in. of R.A.F. 15 section, tested in a 7ft. wide channel and in a 4ft. channel.

Lift coefficient ...	0.15	0.275	0.40	0.475
Seven-foot channel ...	4	7	9	7%
Four-foot channel ...	12	21	27	21%

Such a model would probably never be tested in a four-foot channel, and some doubt may be felt as to the reliability of a correction of such magnitude; but the correction for the seven-foot channel is large enough to receive attention, and I think that the theory may be relied upon for corrections of this size, as it has been found to bring tests of a 6in. × 36in. monoplane in four and seven-foot channels into good agreement.

The interference of the channel walls increases the rate of change of lift with incidence and reduces the stalling angle; reduces the drag by an amount proportional to the lift, giving the drag curve a less steep slope; it also reduces the downwash and so increases the stability and makes the model relatively "nose-heavy."

An inventor recently claimed to have designed a wing which gave very high lift/drag ratio at high lift coefficients, quoting wind channel results in support. I found that his claim was incredible on the basis of the Prandtl circulation theory, but upon ascertaining the dimensions of the model and channel and applying the appropriate correction, the results became more normal. I have dwelt at some length upon this point because of its importance to all who use wind channels, and because the method of correction of results is a recent innovation in this country.

We have also to recognise that the model tests which we make are generally tests upon isolated components which must be applied to complete aircraft with caution. The completest model tests which we are accustomed to use do not represent the aircraft flying under its own power. The action of the airscrew slipstream upon the tail unit is important and highly complex, and I should hesitate to estimate it. The mutual interference of parts is often large, especially when the parts are of low resistance form.

Full Scale Conditions.

The full scale conditions do not call for the same discussion as the model. They are the actual conditions in which we are interested and of which the model conditions are representations. In neither case am I entering into discussion of the accuracy of measuring apparatus. The full scale aeroplane may be employed as a laboratory for analytical observation; the conditions are essentially correct although the analytical method may err, as for example, if the full scale aeroplane be regarded as a means of obtaining the characteristics of a wing isolated in free air. One important point calls for notice. We wish to know the performance of the aeroplane in still air, and errors are introduced by the unknown vertical currents in the atmosphere. A small current introduces a considerable error. The quantities by which the performances of an aeroplane are usually expressed—rate of climb, speed in level flight and gliding angle—are affected by vertical currents, the rate of climb or descent being measured relative to the ground. The quantity which should be measured in order to eliminate the effect of vertical currents is the inclination to the horizontal of the direction of motion of the craft through the air.

We hope by the development of certain apparatus to be able to do this, but at present we rely upon taking a multitude of observations spread over a period, assuming that the average vertical flow is zero. With present methods I do not regard full scale results as reliable to any good degree of accuracy, unless they are based upon a large number of observations.

Experimental Investigation of Scale Effect.

The use of models requires to be based upon an extensive experimental foundation. Until this is achieved it cannot be regarded as better than the best available form of guesswork in the present state of our theoretical knowledge. This experimental foundation has been indirectly built up by the use of models in the development of aeronautical design; but a direct attack by careful systematic experiment has led to more accurate knowledge. The Royal Aircraft Establishment has been engaged for some years upon full scale experiments for this purpose. In the earlier stages the characteristics of various wings were deduced from observations of performance of aeroplanes together with calibrations of the engines and airscrews and model tests of the interaction of airscrew and fuselage and of the drag of struts, wires, undercarriage, etc. More recently the full scale work has been simplified by the elimination of the power unit, thus avoiding a possible source of error and reducing the full scale test to one which can be closely repeated upon the model. The measurements comprise the incidence of the wings and the lift and drag of the complete aeroplane with the airscrew locked in a definite position. The airscrew is stopped in these experiments as the simplest means of reproducing the correct revolutions on the model. The model tested for comparison is as far as possible an exact replica of the aeroplane, the chief difference being the omission of the bracing wires on the model. It is known that there must be a scale effect upon the struts of the model owing to their very small size, but this scale effect is found roughly to compensate for the omission of streamline wires. The fins of air-cooled engine cylinders are not generally reproduced on the model, but care is taken to secure as far as possible the correct air passages round the engine, as this is found to influence the drag considerably. Radiators are reproduced by wire gauze offering the same resistance. Attention to such details in model work is quite important. The elevators of the model are set at the angles which are found to give equilibrium of the air forces on the model about the point corresponding to the centre of gravity of the aeroplane.

In the full scale experiment the quantities measured are the weight of the aeroplane, the indicated air speed, the air density (by pressure and temperature) the rate of loss of height, and the inclination of the aeroplane to the horizon.

The experiment aims at determining the accuracy with which the lift and drag of a complete aeroplane when gliding are predicted by a careful experiment upon a complete model, and from the result we may infer that accuracy with which its performance under its own power would be given by a corresponding model test.

Another line of investigation is by the measurement of the pressure at a large number of points over the wing in flight and in the wind channel. Vertical currents in the air still constitute a source of trouble as the incidence is derived from the speed, rate of change of height and inclination of the aeroplane to the horizon. These measurements enable us to compare lift and centre of pressure, but only the contribution of the normal pressures to the drag.

By flying an aeroplane with the centre of gravity in different positions and altering the tail plane angle, the characteristics of the tail may be deduced from observations of elevator angles and the force on the control column. From this it is possible to proceed to determine the centre of pressure on the wings. Centre of pressure has also been obtained upon an aeroplane with a hinge in the fuselage

by direct measurement of the moment of the forces on the tail about this hinge, and this work has been accompanied by comparative model tests.

Experimental Evidence.

Space does not permit me to review the experimental evidence at all extensively, and I shall only quote some typical results subsequent to the evidence dealt with by the Scale Effect Sub-Committee in 1917. I had hoped when I undertook this Paper to have had available the results of pressure plotting experiments upon a B.E.2c and of gliding experiments upon the Bristol Fighter and three modifications of that aeroplane; but unfortunately the work has advanced more slowly than was expected.

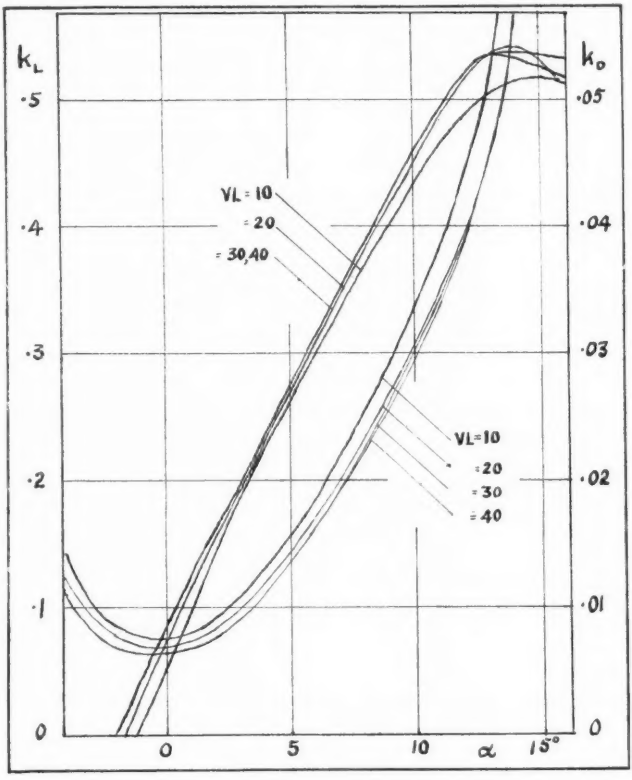


FIG. 5.

Typical scale effect on a thin section aerofoil in the wind channel.

The first set of curves show results of wind channel tests of an aerofoil of aspect ratio 6 at values of VL of 10, 20, 30, 40, L being the chord of the wing. The tests at 20, 30 and 40 were made at one time in one channel upon the same model, but the 10 test was made upon a smaller model in a smaller channel. The scale effect over the whole range is not inconsiderable, but it is becoming small at the upper end. The scale effect is more clearly and strikingly exhibited by the second set of curves which are cross plots at 0°, 5°, 10° incidence and show the ratios of the coefficients of lift and drag to the values at $VL = 30$. It was customary

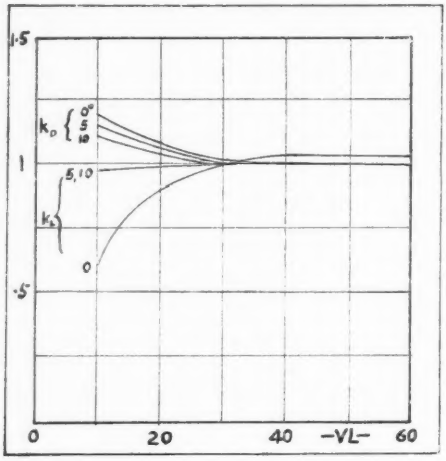


FIG. 6.

Typical scale effect on a thin section in the wind channel.

to test wing sections at $VL = 10$ and these curves show at once that this was not satisfactory; but that the modern practice of tests at $VL = 30$ or upwards, made possible by the use of seven-foot channels, gives results which may be hoped to approximate well to the full scale, a conclusion indicated by the curve for the drag of strut section which I quoted above.

It may be noted that, with the exception of the lift at fine angles, the scale effect upon lift is considerably less than that upon drag.

The results of comparative gliding experiments, full scale and model, which I quote, were obtained upon a B.E.2E aeroplane. The curves are the wind channel measurements of the lift and drag of the complete model corrected for the limitations of the stream. The points are the results of glides and give the lift and drag of the complete aeroplane. These results have been published in R. and M. 762, but were presented there in a different form, and the full scale lifts above 16° incidence have been taken from some unpublished experiments upon the stalling speed. It was previously believed that the full scale maximum lift was in the neighbourhood of 0.6, but the technique of this experiment has been developed recently and both the experiment from which these results are quoted and other experiments upon Bristol Fighters show now a good agreement between the maximum lift full scale and model.

At first inspection the agreement of the model and the full scale is very satisfactory indeed, but further consideration shows that a definite scale effect is indicated. The model differs from the full scale aeroplane by the omission of all bracing wires, and these are estimated to account for a drag coefficient of c.0044 on the full scale. We know, however, that there must be a large scale effect upon the struts, which are very small members. These are estimated to contribute 0.001 to the drag coefficient, and will contribute three or four times as much on the model, leaving, however, a small balance to be explained by some scale effect upon the larger components.

For the sake of including results obtained with a more modern and efficient aeroplane, with wings of the later and commoner section R.A.F. 15, I also reproduce measurements obtained upon the S.E.5A gliding (R. and M. 603), together with comparative model figures one-eighth scale at 80 f.p.s. (R. and

M. 739). The comparison is upon the same basis as before, but the airscrew was stopped in different positions in flight, whereas the model airscrew was fixed with blades vertical. No measurement of incidence was made in flight, and the drag coefficient is therefore plotted against the lift coefficient. The agreement is very satisfactory considering the somewhat less accurate nature of this experiment. In this case the wires contribute about 0.002 to the drag coefficient.

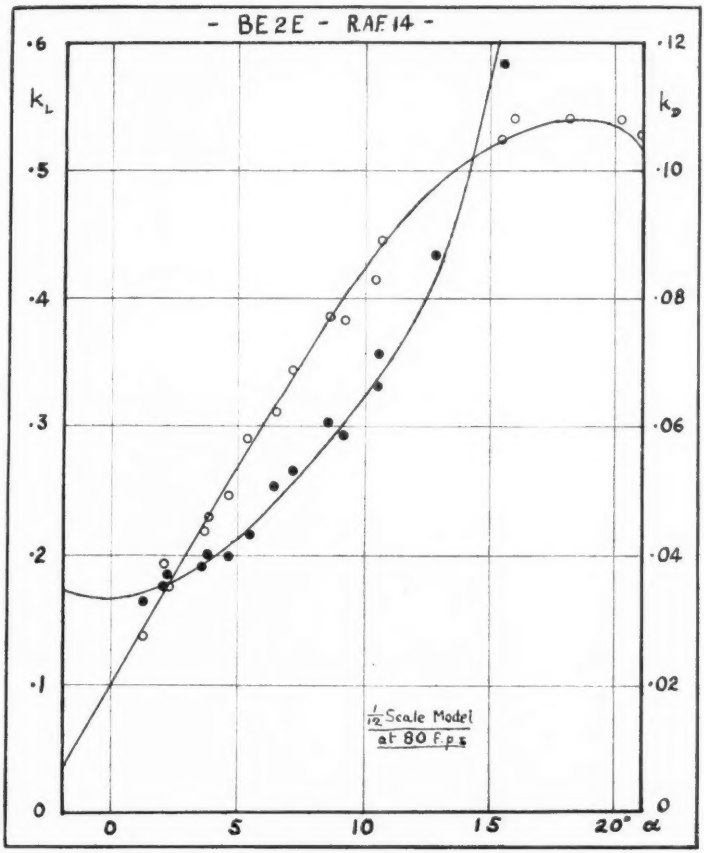


FIG. 7.

Comparison of full scale and wind channel values for the lift and drag of a complete aeroplane. The points were obtained by glides (full scale) and the curves from a test of 1/12 scale model at 80 f.p.s.

The drag of a full scale wing is somewhat less than that of a model tested at $VL = 30$ to 40, and this discrepancy is greatest at the minimum drag (*i.e.*, at fine angles). The reports which the Aeronautical Research Committee have published have indicated that the drag curve has a steeper slope full scale than model. This can be partly, but not entirely, explained by the omission of the correction for the limitation of the stream in the wind channel, which had not been applied in any of that work.

These results are typical of others from which, in my opinion, we may conclude that carefully conducted wind channel tests at sufficiently large values of VL give a very good prediction of full scale performance for the most common type of aeroplane, the thin wing biplane; but we must be cautious of generalising this conclusion.

A highly cambered section, known as R.A.F. 19, has been the subject of extensive experiments. At $VL = 10$ this section gives a remarkably high maximum lift accompanied by a sudden stall with a duality of values of lift over a small range of incidence at the stall, the higher or the lower value occurring according as this region is approached by increasing or decreasing the incidence. At higher values of VL the stall becomes more normal. The full curves show

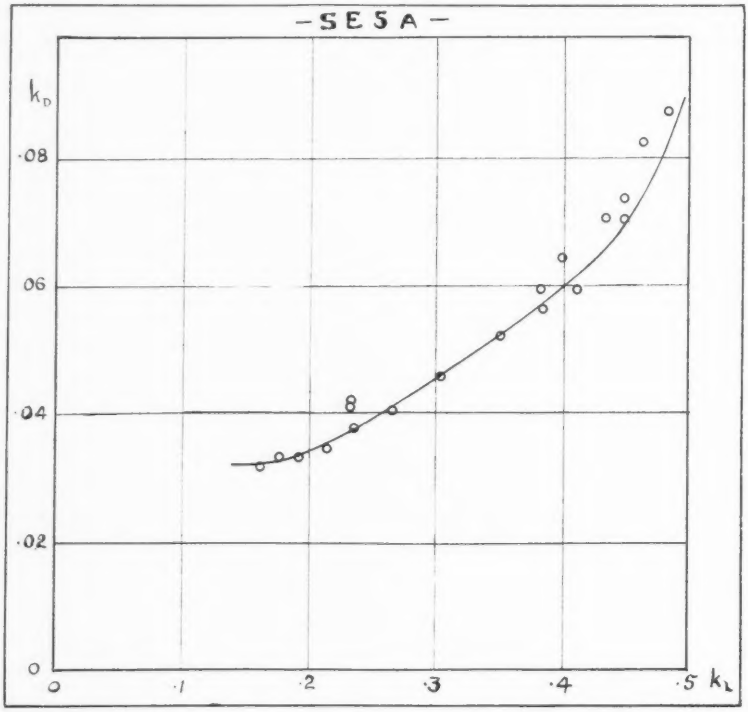


FIG. 8.

Comparison of full scale and wind channel values for the lift and drag of a complete aeroplane. The points were obtained by glides (full scale) and the curves from a test of ⅛ scale model at 80 f.p.s.

the results of model tests at 50, 80 and 120 f.p.s. upon a model B.E.2E with wings of this section. The curves at 80 f.p.s. seem to be reaching some finality of form, but the broken curves giving the results of full scale experiments exhibit a very wide departure from the model in the case of lift. The slope of the full scale lift curve is less steep, and the maximum lift coefficient is only 0.7 as against 0.9 of the model. An endeavour was made to extend the range of the model tests by means of an aerofoil 1.1 feet by 4.4 feet in the fastest channel we have in this country (R.A.E. No. 2·7ft.) and a steady decrease in maximum

lift was obtained. This further test was suggested to me by information I acquired on a visit to Germany. I found that the Göttingen laboratory had made tests up to very high values of VL and had found that the maximum lift of thin or moderately cambered aerofoils increased with VL or became stationary; but that very highly cambered aerofoils exhibited the property we observed on R.A.F. 19. The curves exhibit the variation of the maximum lift of such aerofoils with

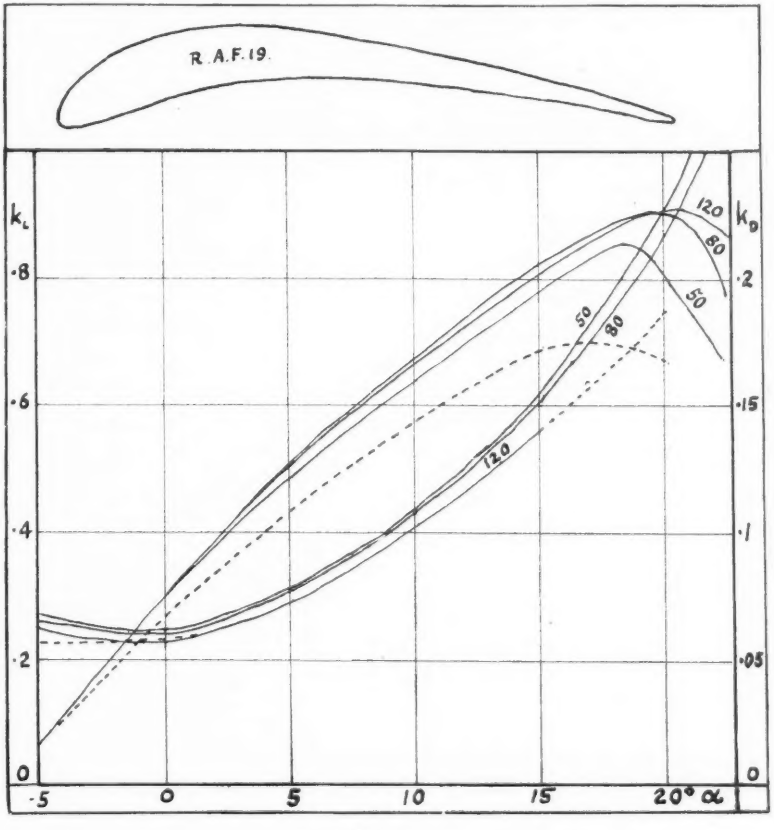


FIG. 9.

Comparison of full scale and model values for the lift and drag of a complete aeroplane. The figures on the curve indicate the speeds in ft. per sec. at which the 1/12 scale model was tested. The dotted curves give the results of glides (full scale).

VL, occurring of course at different angles; the increase of R.A.F. 19 at low values of VL and the decrease of the special high VL test, the B.E.2E model and the full scale, and a Göttingen test upon an aerofoil whose upper surface closely resembled that of R.A.F. 19, although the section is considerably thicker. This unfortunate phenomenon is well established. It seems to be a property of large curvature of the upper surface. The conclusion that slotted wings will behave in a similar manner must not be drawn. Moderately thick wings, such

as section 64, do not exhibit this scale effect, and thicker wings than 64 may attain the lift given by the model.

Fortunately R.A.F. 19, on account of its very high drag, is of no value for biplane construction; but the need for research upon the scale effect on the wings of internally braced monoplanes, in which very thick sections occur, is clearly indicated.

Discussion of the question of scale effect upon the centre of pressure on a wing presents difficulties. From the evidence I have quoted, little scale effect above $VL = 30$ would be expected for thin or moderately thick wings. There is little scale effect on the model above this value and it is difficult to reconcile scale effect upon centre of pressure with absence of scale effect on lift. The

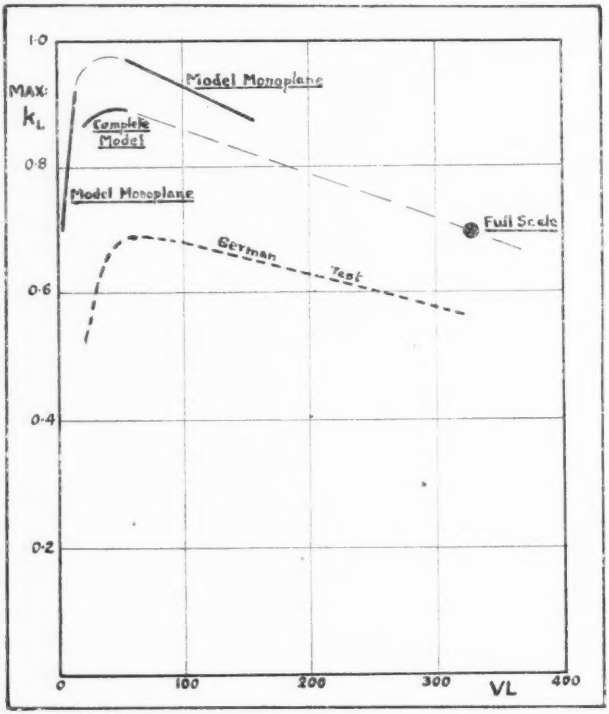


FIG. 10.

Showing the variation of the maximum lift of highly cambered wings with VL.

principal experimental evidence is that obtained by the broken body experiment and this has always given the full scale centre of pressure ahead of the model by from 2 to 5 per cent. of the chord, an amount which has quite a sensible effect upon tail setting and stability. We have been unable to find any reason for error in the experiments, and are seeking information by other means. The comparative pressure plotting experiments upon which we are still engaged should assist in clearing up this question.

Conclusion.

I have endeavoured in this Paper to give a survey of the subject at the risk of some loss of lucidity in the attempt to cover a wide field. Two factors in the

relation between wind channel tests and the full scale seemed to require discussion; the differences between the conditions of the wind channel and those of free flight and the possible results of the failure to conform to the Law of Dynamic Similarity. I have dealt at some length with theories which I think help towards an understanding of the use of model tests, and this has prevented me from examining the experimental evidence at all extensively. Ultimately, we may achieve a complete theory of viscous flow; but this is far off and at present theory does not take us far. In the meantime we must be prepared to find departures of the full scale from predictions based upon the model; but as the experimental evidence has accumulated, the use of model tests has acquired a basis of considerable certainty. Scale effect is seen to be evanescent upon most of the forms which we wish to use, and it is practicable to conduct the model tests upon a scale sufficient to give accurate predictions. On the other hand, we wish at times to use forms, typified by R.A.F. 19 wing, of which this cannot be said, and I am anxious to see a compressed air wind channel built as the most rapid means of advancing the investigation of these shapes; but in any case the comparative full scale and model work should be vigorously continued by the most accurate means we can devise.

Although the relation between model and full scale is not fully understood and the wind channel is not infallible, it is, I think, generally agreed that it has a wide sphere of usefulness, is a method of good reliability, and valuable both as an aid to design and as a means of rapid experimental investigation.

APPENDIX.

Model and Full Scale Airscrews.

Measurements upon model airscrews are useful for testing aerodynamic theories of screw propulsion and for investigating the effects of airscrew on aeroplane and vice-versa. For predicting the performance of the full scale screw a model test is of doubtful value unless the model is elastically as well as geometrically similar and tested at the full scale speed. This requires a channel capable of flying speeds and a motor of considerable power for driving the airscrew. Two additional factors enter into the airscrew problem:—

(1) Twisting of the blades under air forces and centrifugal action.
(2) Compressibility of the air.

The air forces will be the same at the same speed if the airscrews are similar and the ratio of the elasticity of the air to its density is the same. This ratio is equal to the square of the velocity of propagation of pressure waves (a) and the new condition of dynamic similarity introduced is that v/a must be the same. This condition is certainly unimportant up to $v/a = \frac{1}{2}$, but probably becomes important when v/a exceeds $\frac{3}{4}$. The speed of sound (a) is proportional to the square root of the absolute temperature and so does not vary greatly under practical conditions. The stresses due to the air forces therefore vary approximately as ρv^2 and the centrifugal stresses vary as σv^2, σ being the density of the airscrew material. If the material has the same density and elasticity the distortion of the blades is therefore similar at the same speeds.

DISCUSSION.

Mr. A. KIRDANY, dealing with the first part of the paper, said the author had shown a slide with three figures. He would like to know how the middle one was obtained—the one showing the cyclic motion. Was it a mathematical calculation? With regard to Joukowsky's method of getting the proper strength of the circulation to move the stagnation point right to the trailing edge, was that possible in any degree with any angle of incidence, and was there a relation between the angle of incidence and the strength of that circulation?

Colonel DE VILLAMIL said he was a firm believer in the principle of similitude and that consequently he did not agree with the lecturer on some points. He believed that the real reason why there was a difference observed between full scale and small scale experiments was that they were dealing with different forms of motion. In full scale work they were dealing with uniform motion, but in model work they were dealing with *accelerative motion*. If there were no acceleration they would find that dimensional effect disappeared, and they would find that the resistances were equal; he believed they would find that the full scale results agreed absolutely with those of the model.

The principle of relativity is only applicable to the *differential equations* of the second order—d^2x/dt^2, say—and not to the *finite equations*, which are those actually observed and measured. When you integrate, in order to get the finite equations, you have to add constants of integration; and relativity does not *necessarily* hold good for these equations.

Mr. E. RELF said they were all grateful to Mr. Wood for his lecture; he had given them a very coherent account of the relation between model and full scale work at the present time. Dealing with work which was being done at the National Physical Laboratory and which should shed light on the subjects dealt with in the paper, he said that they were investigating the fluid flow round an aerofoil in a wind tunnel, and that the Chairman, Professor Bairstow, was making calculations of the flow round the same aerofoil in a non-viscous fluid. A comparison of the results in the two cases would show where the viscous and non-viscous flows chiefly differed and should be a great help towards understanding the theory of viscous flow. Also they would be able to check the circulation theory and the experiments should give a direct confirmation, or otherwise, of the Prandtl theory. With regard to another point raised by Mr. Wood, namely, the influence of the propeller when comparing the model and the full scale experiments, that was an extremely difficult subject and probably the most complicated that they had to deal with, but that also would be investigated very shortly. They had a model almost ready to go into their large wind channel containing an electric motor of special design, and so they would be able to add horizontal flight and climbing tests to the comparisons seen that evening, and thereby greatly increase their knowledge of the co-relation between model and full scale work.

Mr. H. F. PARKER asked whether there was any reason why a greater value of ρ/μ might not be obtained by using a gas other than air in a closed tunnel. For instance, CO_2 might be used; this gas had a value for ρ/μ of nearly twice that of air. It was easily obtainable and should not be difficult to work with. Another possible gas was chlorine, which had a value three times that of air. This would be objectionable to handle, but if the tunnel was properly sealed and reasonable precautions taken useful results might be obtained. There were other heavy gases that might be used in this connection. Two cases might be considered; first, that in which a gas having a value for ρ/μ greater than air was used at atmospheric pressure to partially bridge the gap between present model and full scale results; and second, the possibility of getting full scale values from models by compressing such a gas. In the latter case a special tunnel would have to be constructed to resist considerable pressures. These pressures would be much less than those necessary to secure similar results in a compressed air tunnel and the question that arose was whether the simplification in construction compensated for the difficulties of working with undesirable substances. In the case of the tunnel at atmospheric pressure, however, it might be possible to secure results of considerable value for comparatively little trouble. A closed circuit tunnel would be necessary—there was one of that type, eight feet in diameter, at the Washington Navy Yard, but he was not familiar with the detail construction of the large tunnels in this country, though he believed most of them to be of the open end type. However, such a tunnel could be made entirely gas tight

with little difficulty, and as the observers would be outside working in pure air they would run little risk. The troublesome operations would be those of filling the tunnel and of setting the model in position. Provided precautions were taken to remove or protect anything likely to be attacked, chlorine might actually be safer than CO_2 as observers could smell any leak long before there was enough of the gas in the air to harm them. The absence of a warning of this nature might make CO_2 more dangerous.

The CHAIRMAN said he remembered, not very many years ago, that there was much disagreement between measurements of model and full scale experiments. One of the points always to be remembered when comparing model and full scale experiments was the degree of accuracy of the experiments. So far they seemed to have removed a great deal of the error of observation, and he, with Mr. Wood, would like to see a great deal more attention paid to the co-relation of model and full scale work. But it was when Mr. Wood tried to interpret scale effect in terms of fluid motion that he (the Chairman) found himself unable to follow him; the existence of the circulation theory was a perpetual trouble of his. Dealing with the work being carried out by himself and others with regard to the application of Prandtl's theory of non-viscous fluids to aeronautics, there was little doubt that they would get a solution to compare with experiments, but he did not expect to find the circulation at all in the experiments. It seemed to him that viscous fluid motion had not got circulation in it. It was difficult to know why the circulation theory accounted for so much as it did. In a paragraph in the paper, Mr. Wood had said, "On the assumption that the flow is steady laminar flow the velocity gradient and the drag exerted on the surface can be calculated." By "laminar flow" he (Mr. Wood) meant the conventional flow, *i.e.*, the flow parallel to the plate itself. But later, at the end of the paragraph, the author seemed to have identified laminar flow with steady flow, because he said, "The state of steady flow occurs, however, only at very low values of $vl\rho/\mu$, and the values with which we are concerned in aeronautics are well inside the range of turbulent flow." He (the Chairman) did not think there was any connection between laminar flow and steady flow, and he did not think it was a well-founded statement, that the values with which they were concerned in aeronautics were well inside the range of turbulent flow. His own impression was that until they passed the stalling angle of a wing the flow was steady. He did not say that it was any better established than the contrary hypothesis, but he thought they could hold the opinion on present evidence. If they took the case of a flat plate which was not infinite in length, then obviously they could not get laminar flow. Suppose the fluid flowing over a plate of finite width but infinite length. There was no friction on the plate until the particles actually came into contact with it, and then the intensity of the resistance suddenly became very big, so that there was a slowing up of the streamlines and a gradual widening. Certain calculations of flat plates where $vl\rho/\mu$ was as great as 10,000 had just been completed, in which the flow was still steady, and his own impression was that for skin friction on a thin flat plate values of $vl\rho/\mu$ as high as we could reach were still consistent with steady motion. The whole subject was one of great difficulty, but one in connection with which we were beginning to see some light; whether or not the Prandtl theory could be accepted at the moment as anything more than empirical formulæ did not eliminate its value in aeronautics. He anticipated that it would be put on some better fundamental basis before many years were passed.

Mr. MCKINNON WOOD, replying to the discussion, said he was not quite sure whether he was brave enough to reply to Mr. Kirdany, because he did not pretend to be a mathematician. Mr. Kirdany had asked how the circulation in the middle picture was calculated. Actually the figures were copied very freehand from a book. (Laughter.) He understood, however, from mathematicians that it was possible to calculate lines like that. In the calculation they did not know at the start for certain what sort of aerofoil they would end up with; but they ended with a shape, and if it were not the shape they wanted they tried again. It was possible to calculate forms of flow round bodies both in translation and in

circulation and to superimpose them, and by doing so in the right proportion they would get the stagnation point to come to the trailing edge at any angle of incidence. Colonel de Villamil, he gathered, had suggested that the scale effect that they observed from the wind channel to the full scale was due to the conditions of experiment being different. That was a point to which he (Mr. Wood) had referred and it was one they had always to bear in mind. The air in the wind channel was not the same as the air through which the aeroplane flew. Neither were absolutely uniform motions. In the air there might be a certain amount of turbulence, and probably a good deal more in the wind channel, and it had been found that, by introducing artificially a high degree of turbulence in front of fine shaped bodies in a wind channel it was possible to affect the drag quite considerably, but he did not know of any evidence that quite a large amount of turbulence in the channel affected seriously the lift and drag of an aeroplane wing. He was speaking throughout the paper rather from the point of view of aeroplane design than such things as airship resistance. He knew from experience that measurements of the drag of airships in different channels did not always give quite the same result. Another interesting thing he might mention was that at the R.A.E. they had measured the drag of a section such as the strut section which he had shown first. It was a blunter section than the one shown, a proposed section for an aeroplane streamline wire, and the scale effect on it was very large and very erratic. In trying to make measurements in different channels they had found they could not repeat them; the results were entirely different, so that where they had a flow which was critical—where they were in the region of large scale effect (if there was such a thing as scale effect)—then he believed turbulence in the air might make a great difference to the results obtained.

With regard to the use of CO_2 or chlorine or other gas in wind channels, Mr. Relf and he had both looked into that question, and had come to the conclusion that there was no real reason to prefer any other gas or liquid to air, which had obvious advantages from the point of view of convenience. The thing could be made to work with air, although considerable difficulties would be met.

At the last meeting of the Society he had heard the Chairman give his views on circulation and the Prandtl theory, and he (Mr. Wood) considered that it was very important to carry out the experimental work to which Mr. Relf had referred; it was only by carrying out experiments that they would find out whether or not there was circulation round the wing. He believed that sufficient measurements had been made of the flow round the wings of an aeroplane to indicate that the circulation which Prandtl supposed to be there was actually there. They had measured the direction of flow in front of and behind wings and off the tips, and except in the case of the measurements behind, where the calculations by Prandtl were not made to take into account the rolling up of the vortex sheet into two main vortices, the agreement between the measured values and those calculated by Prandtl were extremely close, and in the case of the measurements behind the wing the experimental value lay between two hypotheses.

He took it, when preparing the lecture, that there was very little hope held out in the chapter on scale effect in the Chairman's book on "Applied Aerodynamics." The law of dynamic similarity was stated, but there seemed to be nothing suggested which would justify them in using wind channel experiments and applying them to the full scale, and he had wondered why the Chairman should always have been so enthusiastic in insisting on the value of model work. He admitted that the whole of his theoretical arguments in the paper were extremely shaky; but he felt that there was a certain amount of reason in them, and he liked to see some glimmer of light on the subject suggesting that there was a theoretical justification for using wind channel results. On the question of steady and laminar flow, he thought when he wrote the paper that he was practically quoting from the Chairman's book. (Laughter.) He wrote it, perhaps, after reading the chapter rather late at night. Perhaps if he had said that the state of laminar flow occurred only at very low values, that might be a correct statement. He was talking about thin plates. He was thinking of the

state of flow in which they had resistance proportional to the speed and the state when they got the resistance more or less proportional to the square of the speed, which was the state they had in actual aeronautical problems.

At the conclusion of the meeting a very hearty vote of thanks was accorded Mr. McKinnon Wood for his interesting and lucid paper.

Professor B. MELVILLE JONES (*communicated*): I regret that I shall not be present to take part in the discussion of Mr. McKinnon Wood's interesting and comprehensive paper. I would like, however, to be allowed to lay emphasis on the great value to aeronautical progress of the work that has been done by Mr. McKinnon Wood and others at the R.A.E., in the direction of developing the technique of free flight experiments up to a point where sufficient accuracy can be attained to allow the results to be used as a direct check upon the value of model experiments.

It may be hard for anyone who has not tried it to realise the extreme difficulty of obtaining accurate results from experiments in free flight. I have had some experience of this type of work myself and I am convinced it will be found impossible to obtain any results whatever of the accuracy required in this connection without employing a staff who, besides being very capable experimenters, have had a wide and long experience of the peculiar difficulties to be overcome in this class of work.

In the absence of some all embracing theory whereby aeronautics could be freed from its present reliance on masses of experimental data—and there is no sign at present of such a theory appearing on the scientific horizon—there is nothing more certain than that real advances in aeronautics will depend to a large extent upon wind channel data. More or less exact quantitative data as to the air reaction on *separate* parts is an essential to progress, and if this cannot be obtained from theory on the one hand, or on the full scale on the other hand, there is no other source available than the wind channel or some equivalent small scale experiment. Full scale experiments are almost incredibly costly, when every source of expense is taken into account, so that—even apart from their extreme difficulty—they cannot be used to provide data of the immense variety that is required in aeronautical design.

The bulk of the empirical data required must, under these circumstances, be obtained on a small scale, but Mr. Wood has explained in his paper that we have no theoretical justification for supposing that the small scale data bears any relation whatever to the full scale. There is therefore only one course open to us and that is, whilst obtaining the bulk of our data on the small scale, to find out by a few well chosen full scale experiments those classes of data for which model experiments do represent the full scale, and in particular those classes in which they do not.

If the above statement of the position is correct it is obviously of vital importance to keep going the organisation whereby this full scale checking can be done, because without it we are building on sand, since at any time we may suddenly realise that the data on which we have been relying for some advance in design does not represent the facts. In the absence of some such organisation, whereby accurate full scale experiments can be made, we are in fact reduced to waiting for costly experiences of error in the prediction of the behaviour of new types to awake a suspicion in our minds that there is something wrong with our data. It takes, however, many such experiences to give us any feeling of certainty that there is an error and to indicate where the error lies.

The moral of all this is the absolute necessity of maintaining the experimental research staff at the R.A.E. and particularly that part that deals with full scale work, at the highest state of efficiency possible, having regard to the financial position of the country. My main object in making this contribution to the discussion is to emphasise this point at a time when severe financial difficulties make it important to distinguish clearly what branches of research are essential to progress on sound lines.

LIBRARY.

List of Current Periodicals.

Members may be interested in the following list of periodicals which are received regularly and placed on the Library table:—

TITLE.	COUNTRY.	CHARACTER.
Advisory Committee for Aeronautics. Report	England	Irregular.
Aerial Age	America	Monthly.
Aero-club Argentine. Bulletin (El Avion)	Argentine	Monthly.
Aero-club de Espana. Boletin (Oficial)	Spain	Quarterly.
Aero-club de Portugal. Boletin (Revista Aeronautica)	Portugal	,,
Aerodynamic Institute of Aachen. Abhandlungen	Germany	Irregular.
Aeronautical Abstracts	England	Monthly.
Aeronautical Research Committee. Reports and Memoranda	,,	Irregular.
A.R.C. Internal Combustion Engine Sub-Committee Reports	,,	,,
L'Aeronautique	France	Monthly.
L'Aerophile	,,	,,
Aeroplane, The	England	Weekly.
Les Ailes	France	,,
L'Air	,,	Fortnightly.
Air League Bulletin	England	Monthly.
Aircraft	Australia	,,
L'Ala d'Italia	Italy	,,
Automobile Engineer, The	England	,,
Aviation	America	Fortnightly.
Bulletin de la Federation Aeronautique Internationale	France	Quarterly.
Canadian Patent Office Record	Canada	Weekly.
Cassier's Magazine. (See Engineering and Industrial Management.)		
Civil Aviation Communiques	England	Irregular.
Conquete de l'Air	Belgium	Weekly.
Coventry Engineering Society Journal	England	Monthly.
Deutsche Luftfahrer Zeitschrift	Germany	,,
Engineer, The	England	Weekly.
Engineering	,,	,,
Engineering and Industrial Management. (Cassier's)	,,	Fortnightly.
Ergebnisse der Arbeiten des Aeronautischen Observatorium	Germany	Irregular.
Export World and Commercial Intelligence. (Aeronautics)	England	Monthly.
Faraday House Journal	,,	,,
Flight	,,	Weekly.
Flugsport	Germany	Fortnightly.
Gionalle dell'Avazione	Italy	Weekly.
L'Indicateur Aerien	France	Monthly.
Institute of Engineers and Shipbuilders of Scotland	Scotland	Irregular.
Letectvi	Cheko-Slovakia	Monthly.
Lot	Poland	,,
Luftweg	Germany	,,
Mechanical Engineering	America	,,
Metal Industry	England	Weekly.
Meteorological Office. Professional Notes	,,	Irregular.
Meteorological Society, The Royal. Official Journal	,,	Quarterly.
Monthly Air Force List	,,	Monthly.
Monthly Weather Bureau	America	,,
National Advisory Committee for Aeronautics. Reports	,,	Irregular.
North-East Coast Institute of Engineers and Shipbuilders	,,	Annual.
Rassegna Marittima Aeronautica Illustrata	Italy	Monthly.
Rendiconte dell'Istituto Sperimentale Aeronautico	,,	Alt. Months.
Revista Aeronautica	Portugal	Quarterly.
Revue de L'Aeronautique Militaire	France	Fortnightly.
Revista Mensile del Touring Club Italiano (Vie d'Italia)	Italy	Monthly.
Royal Engineers' Journal	England	,,
Royal Institution. Proceedings	,,	Annual.
Royal Meteorological Society's Journal. (See Meteorological.)		
Royal Society of Arts Journal	,,	Fortnightly.
Royal Society of Edinburgh. Proceedings	Scotland	Annual.

TITLE.	COUNTRY.	CHARACTER.
Royal Society of New South Wales. Transactions	Australia	Annual.
Sea, Land and Air	,,	Monthly.
Society of Automotive Engineers	America	,,
Steel Structures	England	Quarterly.
Svensk Motor-Tidning	Sweden	Fortnightly.
Technical Memoranda	England	Irregular.
Technical Notes	America	Monthly.
Technique Aeronautique	France	,,
Technical Memos	England	Irregular.
United Service Institute of India	India	Quarterly.
United Service Magazine	England	Monthly.
United States Air Service	America	,,
Vliegveld, Het	Holland	,,
Wireless World	England	,,
World's Carriers	,,	,,
Zeitschrift fur Flugtechnik	Germany	Fortnightly.

REVIEWS.

L'Aeronautique des Origines à 1922. Comte de la Vaulx, Paul Tissandier, Charles Dollfus.

It is a matter for increasing regret that the study of aeronautical history—briefly, "the evolution of flight"—has not hitherto received more serious attention. In recent years aeronautical science has achieved results which are ultimately destined to affect profoundly civilisation at large, and yet the number of those interested in the earlier history of the science still remains strangely limited. In England, at least, scholarship has played no part in its elucidation, and such compilations as have appeared have been largely drawn from secondhand and usually unreliable authorities. Two reasons may be offered for this apparent neglect. Until times within memory aeronautical projects were commonly regarded as the work of mistaken enthusiasts or cranks and dismissed accordingly; and since the days of achievement men have been too fully occupied in doing to find time or inclination for the study of earlier and apparently fruitless periods. On that account the majority of readers of this journal, concerned mainly with matters technical, may find the work under review less interesting than it will be to collectors of the raw materials of aeronautical history.

France, it is true, has been somewhat better served, and fittingly so, having regard to the high place she holds in aeronautical invention and development. Moreover, it may be hoped that the small but scholarly work of Giuseppe Boffito, "Il Volo in Italia," published in Florence last year, is significant of a more serious attitude now becoming apparent.

In any case the appearance of this handsome volume is welcome, and its distinguished authors deserve praise as much for the admirable spirit which prompted it as for the enterprise and skill displayed in its production. The publication was conceived by that veteran in aeronautics, Comte de la Vaulx, as a tribute to M. F.-L. Bruel, whose great work, "Histoire Aéronautique par les Monuments," published in 1909, it is designed to supplement. Following the lines of Bruel's earlier work, the present volume consists of over 100 excellent reproductions, wholly in facsimile and many in colour, of engravings, drawings, portraits, autographs, and so forth, varying of course both in interest and historic importance, but accompanied by ample descriptive notes. Rather more than half the plates illustrate the earlier phases of ballooning, the endeavours to construct dirigibles and some forerunners of mechanical flight, while the last forty deal with the period of conquest, "Le Triomphe de l'Aéronautique," as well as a few connected with the Great War. Space forbids more than brief mention of a few at random.

Pilâtre de Rozier's first free ascent—a wondrous achievement in its time—is seen in a contemporary drawing by Desrais, while another excellent drawing depicts the ascent of the huge "Flesselles" balloon, with Pilâtre portrayed—truly, as one can well believe—balancing himself on the gallery. The plate of Garnerin's parachute recalls that his first descent was regarded by Wilbur Wright as the most courageous act in aeronautical history—an opinion expressed long ago by Sir Sydney Smith, who went to see Garnerin's descent in London in order "to shake hands with a brave man." The engraving of the rescue of Zambeccari—the unsuccessful rival of Lunardi and a remarkable pioneer of ballooning—is likewise associated with a modern pioneer, for the youthful imagination of Lilienthal was first quickened to an interest in aeronautics on reading the adventures of Zambeccari. The fine portrait of Charles Green—a coloured design of

his famous "Nassau Balloon" forms a decorative wrapper for the book—confirms an impression of the sturdy character of this life-long exponent of the free balloon, though incidentally it is not a fact (as the text states) that his career was "without accident." He had several notable escapes from imminent death when his admirable judgment and self-control alone saved him. The crude poster of Gale's balloon with an upper and lower car connected by a rope ladder, is less interesting than the fact that Coxwell afterwards used it in Berlin (about 1848) to demonstrate that "aerial torpedoes" could be effectively dropped from a balloon. The countless early projects and endeavours in the direction of "dirigibles" are illustrated in several plates—notably the "Eagle," designed by Count Lennox in 1832, which it is interesting to compare with his later and even less successful London venture. Of greater importance are the designs of Pierre Julien, 1851—the notable streamline form of his "Précurseur" justifies the name—Giffard's steam-driven dirigible of 1852, and the famous "La France" of Renard and Krebs; though one might wish that the latter plate depicted less landscape and more dirigible.

Of "heavier-than-air" design there is naturally little in the pre-conquest period. The large picturesque lithograph of Henson's "Aeriel"—which, as a print, is better in colours—gives rise to a reflection as to how far Henson was indebted to Cayley, and from that to the more certain conviction that the significance of Cayley's ideas and calculations on dirigibles, and his experiments with gliders in relation to mechanical flight have not yet been justly appraised. In any case it is misleading to refer to Henson as the "constructeur," for the "Aeriel" was never constructed save on model scale. A more controversial issue is raised by the plan of Ader's "Avion," though Count de la Vaulx, in his introduction, definitely claims for his compatriot "la gloire impérissable d'avoir, le premier, réalisé le vol humain." Nor is it wholly profitable to debate such claims—honour in full measure may justly be allowed to the unsuccessful and successful pioneers alike, and we may be satisfied to believe that the latter would be the first to accord it in full measure to their forerunners. The later plates in the periods of "Conquest" and "War" do not call for comment—the names of Lilienthal, Ferber, Latham, Wilbur Wright, Pégoud and Guynemer, are in themselves splendidly sufficient.

As to the text, it would be ungracious to criticise the Count de la Vaulx's "Introduction"—if as to matter it is neither more nor less adequate than many similar pieces of writing, it is certainly inspired with a fine spirit of enthusiasm and generous appreciation. But one may regret that it is marred even by such small errors as the reference to "Pierre Wilkins" (the hero of Paltock's charming romance) as an author, or the repeated mis-spelling of the name of Sir George Cayley. The *format* of the volume has only been equalled in Bruel's own book, possessors of which may wish that a uniform size had been adopted. Moreover, users of the present volume as a work of reference will doubtless also wish that some more convenient arrangement of text and plates (the latter being entirely unnumbered) had been followed. Such defects detract, however, but little from the historical value of the book, or from the pleasure it must afford to the fortunate but limited owners of it. Should it serve to inspire some English publisher to undertake a similar enterprise, it will have served a double purpose.

The Internal Combustion Engine. By Major H. E. Wimperis.

The author has provided a useful textbook for students, giving in a small volume a survey of the theoretical and practical problems of internal combustion engines.

He has been particularly successful in putting forward the essential subjects from the various sciences necessary for a clear understanding of the working of the internal combustion engine.

The wide subject covered makes it impossible to go very deeply into all aspects of the problems involved, but references to numerous papers dealing with particular subjects are given.

The book is divided into three approximately equal sections dealing with (1) thermodynamics, (2) gas engines and gas producers, and (3) oil and petrol engines respectively.

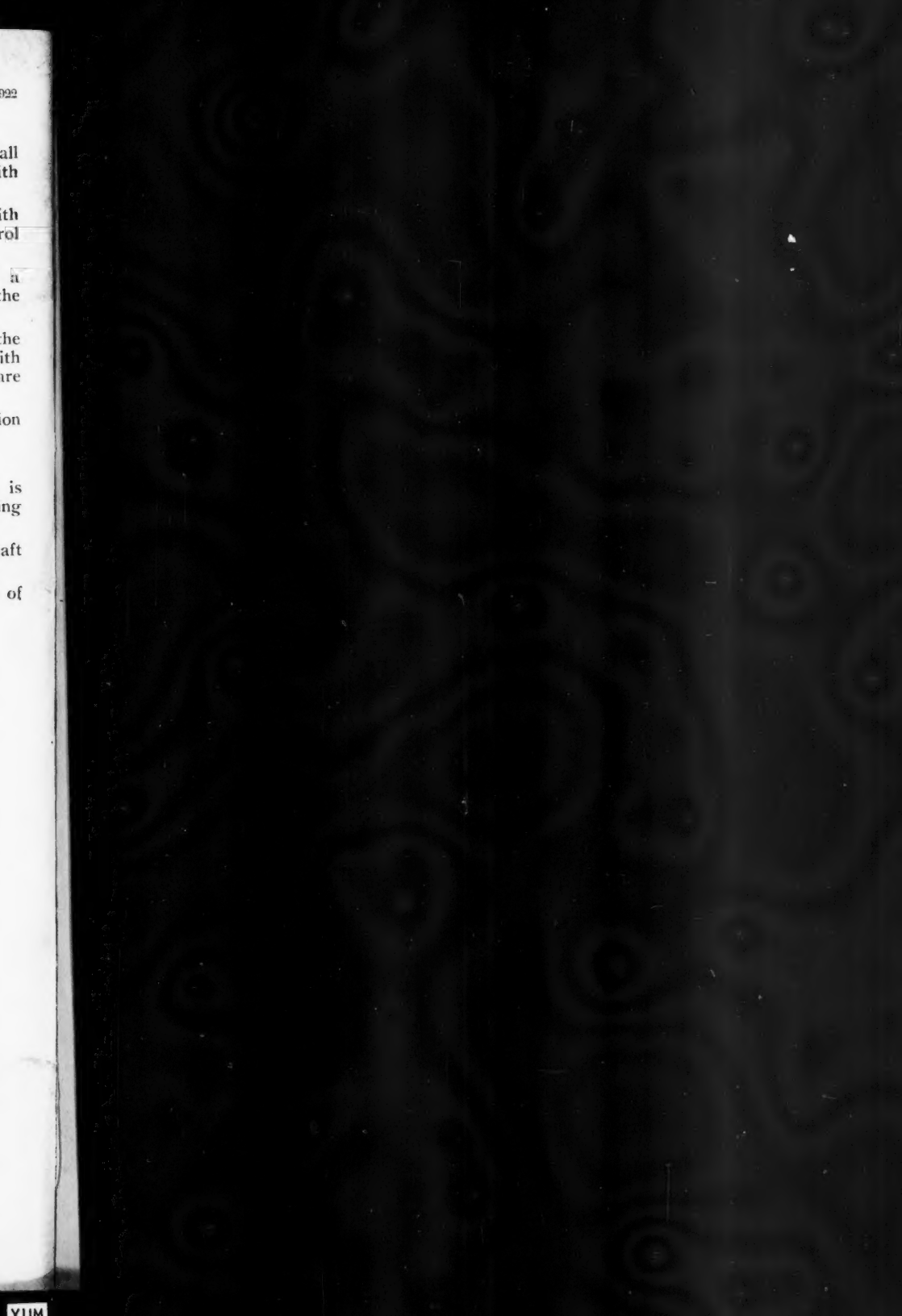
In Section I. the laws of gases and the various cycles are explained in a simple manner. A somewhat large proportion of the space is devoted to the researches of various workers on explosions of still gases in closed vessels.

In Section II. descriptions of various types of gas engine, including the gas turbine and the Humphrey pump, and gas producers are given, together with test results of actual plants. A few problems of design, such as balancing, are discussed.

In Section III. various types of oil and petrol engine, carburettors and ignition apparatus are illustrated and described.

Very little space is devoted particularly to engines for aircraft.

In common with many textbooks on engines, very little consideration is given to the methods of the designer in arriving at a solution of his conflicting theoretical and mechanical problems.

A discussion on the art of mechanical design as applied, say, to an aircraft engine would form an interesting addition to the book.

Some excellent examples of exercises for the student are given at the end of each chapter.